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MATHEMATICS FOR FRESHMEN STUDENTS OF ENGINEERING

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MATHEMATICS FOR FRESHMEN STUDENTS OF ENGINEERING

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THEODORE LINDQUIST

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MATHEMATICS FOR FRESHMEN STUDENTS OF ENGINEERING

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CHAPTER I.

HISTORICAL SUMMARY.

I. PERIODS OF ENGINEERING HISTORY.

The history of engineering education in the United States falls into three periods separated by two notable events. The first of these is the establishment, in the first part of the nineteenth century, of courses in civil as distinct from military engineering; the second the passage of the Land Grant Act in 1862. While in this case, as in many others, succeeding periods blend into each other without any sharp line of demarkation, yet these three periods are very real and distinct.

II. FIRST PERIOD—TO 1824.

During the first period there were two sources of engineering education; the apprenticeship system and the United States Military Academy.

1) The Apprenticeship System.

The apprenticeship system was confined chiefly to New England; it continued in existence until the Civil War and furnished most of the engineering education up to about 1835.

Instruction consisted of such information as the apprentice could "pick up" through questions in connection with field and office work, for which privilege he paid one hundred dollars per year for a term of three years. For his work in the field he received twelve and one-half cents per hour as compensation, but before the beginning of the Civil War nothing was paid, as a rule, for office work. After that date all work was paid for at the rate mentioned. The above conditions were quite uniformly prevalent. This system presented the extreme of "practical" instruction to the exclusion of all theory.

2) The United States Military Academy.

The United States Military Academy was one of the main sources of engineering education during this period since many of its graduates took up work along civil lines, and remained a factor in engineering education until 1850.

As early as September, 1776, a committee of the Continental Congress recommended a bill for a "Continental Laboratory and Military Academy." Later Washington, as president, several times urged Congress to establish such an institution. Nothing was done, however, until 1802 when the bill authorizing its establishment was passed, March 16. July 4, of the same year, the Academy was opened at West Point with ten cadets.

At first the instruction in mathematics was very limited extending only through Hutton's first volume. By 1810 instruction included arithmetic, logarithms, elementary algebra, geometry, trigonometry, mensuration of heights and distances, planimetry, stereotomy, surveying and conic sections. In 1816 a four years' schedule of studies was adopted in which the mathematical work "embraced the following branches, namely: the nature and construction of logarithms, and the use of the tables; algebra, to include the solution of the cubic equation, with all preceding rules; geometry, to include plane and solid geometry, also ratio and proportion, construction of geometrical problems, application of algebra to geometry, practical geometry on the ground, mensuration of the planes and solids; plane trigonometry, with the application to surveying, and the mensuration of heights and distances; spherical trigonometry, with its application to spherical problems; the doctrine of infinite series; conic sections

with their application to projectiles; fluxions to be taught at the option of the professor and students." "Fluxions," however, were seldom given. This curriculum also included "A complete course in engineering."[1] From 1816 there has been a gradual growth in the amount of mathematics offered, up to the present schedule, which was put into operation with the class entering March, 1908.[2]

III. CONDITIONS IN ENGINEERING AT THE TRANSITION FROM THE FIRST TO THE SECOND PERIOD.

A brief glance at conditions in the engineering world about the time of transition from the first to the second period may be of interest.

1) Development of Natural Resources.

The output of iron had risen from 54,000 tons in 1810 to 165,000 tons in 1830 and to 347,000 tons in 1840. Coal mining was begun about 1820 with an output of 65,000 tons for that year. Progress in agricultural developments gave rise to the manufacture of farm implements; plows and mowers, of quite good pattern, were produced as early as 1820.

2) Transportation—(*i*) *Canals.*

The first attempt at a solution of the transportation problem in this country was the building of canals. As early as 1793 a short canal was constructed in order to make a contour around the falls in the Connecticut river at South Hadley, Massachusetts. The first freight and passenger canal, which was three miles in length, was begun the same year and finished in 1804, but has not been in use since 1850. The principal canal building era lasted from 1810 to about 1840. Only one canal of importance, the Illinois and Michigan, in course of construction from 1836 to 1848 was finished after this time.

[1] Bvt. Maj.-Gen. G. W. Cullum, Biographical Register of the Officers and Graduates of the U. S. Military Academy with the Early History of the United States Military Academy. Boston and New York, 1891.

[2] Chap. II, p. 41.

(*ii*) *Railroads.*

Canal building decreased as a result of the coming of the railroad. The first railroad, three miles in length, was constructed in 1826 at a cost of $34,000 and used horses as motive power. In 1829 the first English made locomotive was imported and used on a sixteen mile track. A year later the first American locomotive was constructed. The second American locomotive was used by the South Carolina Railroad which ran between Charleston and Hamburg. On its first passenger trip, made January 15, 1831, it attained a speed of 15 to 20 miles per hour. The following table gives the mileage of railroads up to 1900:

1830	23 miles
1831	95 miles
1832	229 miles
1835	1,098 miles
1840	2,818 miles
1845	4,633 miles
1850	9,021 miles
1860	30,626 miles
1870	52,922 miles
1900	190,082 miles

(*iii*) *Steamboats and Bridges.*

The application of steam to water traffic was made by Robert Fulton with his Clermont, as early as 1807. It was not used for transatlantic ships, however, before 1839. The building of tunnels and bridges which had just begun at this time also greatly faciliated traffic.

IV. SECOND PERIOD—1824 TO 1862.

During the first years of the nineteenth century a great step forward was made in education in the United States by the introduction of laboratory instruction. The principle involved was further developed in the organization of courses giving work in civil engineering. The Rensselaer School, founded by Stephen Van Rensselaer in 1824, was the pioneer in this movement. The date of its founding, therefore, serves to indicate the beginning of the second

period, although this period was not fully established until some years later. Following the establishment of the Rensselaer School several of the literary institutions then existing organized courses in engineering. As a consequence there are to be found two distinct classes of technical institutions which will here be considered briefly; purely technical institutions and literary-technical institutions.

1) Purely Technical Institutions.

(*i*) *Rensselaer Polytechnic Institute.*

The Rensselaer School was the only purely technical institution founded during this period. Concerning its object, Stephen Van Rensselaer wrote Rev. Samuel Blatchford of Lansingburg, October 5, 1824, that he had established a school for those "who may choose to apply themselves in application of science to the common purposes of life. My object is to qualify teachers for instructing the sons and daughters of farmers and mechanics, by lectures and otherwise in the application of experimental chemistry, philosophy and natural history to agriculture, domestic economy, the arts and manufacture." The institution has, however, devoted itself to the direct preparation of its students for technical occupations. The original name, Rensselaer School, was changed in 1833 to Rensselaer Institute. Instruction in civil engineering was first offered in the annual announcement for 1828 but no special course was established before 1835. The institution was again reorganized in 1851, when the present name, Rensselaer Polytechnic Institute, was adopted. At this time the course of study was also extended from one to three years. In 1852 there was added a preparatory course of one year, which was abolished in 1862 when the course was lengthened to four years.

In the mode of instruction the Rensselaer School made a very decided departure from that of the other institutions existing at the time of its organization. The mode was in substance as follows: each new topic was taken up by the professor in charge in a lecture upon which the class was quizzed the following day, the class was then divided into sections which met in various smaller rooms. At the meetings of each of these sections a student would be selected to repeat the lecture of the day previous. Although an increased number of students has compelled a change in the form of carrying on the work to that of the lecture and quiz, still the spirit of the orig-

inal mode has been retained. This spirit—that of "Learning by doing"—is the feature to be emphasized in connection with this institution.[3]

The following is an outline of the course of study offered in 1854 after the course in civil engineering had fully crystalized. It is to be regretted that the records do not show the number of exercises devoted to each subject.

First Year.

First Term—Algebra, geometry, general physics, graphics, geodesy, English composition, French.

Second Term—Trigonometry, higher algebra, general chemistry, graphics, geodesy, botany, English composition, French.

Second Year.

First Term—Analytics, differential calculus, practical trigonometry, general physics, minerology, chemistry, descriptive geometry, English composition, French.

Second Term—Integral calculus, general physics, geology, zoology, graphics, geodesy, English composition, German.

Third Year.

First Term—Mechanics, astronomy, physical geography, geology, industrial physics, English composition, philosophy.

Second Term—Construction, mechanics, mining, geodesy, practical astronomy, graphics, metallurgy, industrial physics, philosophy.

At first those "who have a good knowledge of arithmetic and can understand good authors readily" were received into the institution. The requirements were raised, however, until at the time the above schedule of studies was put into operation they were, in mathematics, arithmetic, including the metric system, plane geometry and algebra to equations of the second degree. In view of the fact that a preparatory course of one year was offered at this time and that the college course was only three years in length these were really higher than the actual requirements for entrance to the institution. Davies' Legendre's Geometry, Davies' Bourdon's Algebra, Chauv-

[3] Palmer C. Rickets, History of the Rensselaer Polytechnic Institute 1824-94. New York, 1895.

enet's Trigonometry, Church's Analytical Geometry and Church's Calculus were the texts used. By 1893 the work in mathematics had been changed to solid geometry, trigonometry and algebra for the first year, analytical geometry for the second, and the calculus for the third. Two years later the differential calculus was put into the second year.[4]

2) Literary-Technical Institutions.

About the middle of the nineteenth century several literary institutions recognized the advisability of adding courses in engineering. As a rule these were at first a modification of the final two years of their existing courses, but they gradually developed into the highly specialized ones of the present. Because of the similarity of development, changes in the schedule will be given fully for two institutions and only special characteristics of the others.

(*i*) *Union College.*

Union College, at Schenectady, N. Y., was established as early as 1795, but the Department of Civil Engineering was not organized before 1845. Until 1852 the work required in mathematics was algebra (Bourdon), one year; plane geometry (Legendre), one-third year; solid geometry, one-third year; plane and spherical trigonometry, one-third year. Beginning with 1852 one-third year was added in each of the following: algebra, conics (Jackson), analytic geometry of three dimensions (Davies) and the calculus (Davies). A little later the course was changed so as to call for only two years, but the entrance requirements were raised so as to include a large amount of work previously given in the four years' course. The entrance requirements in mathematics were then two terms each of algebra and geometry, and one term each of plane and spherical trigonometry and geometrical drawing. During the first year of the two year course one term was given to each of the following: analytic geometry, accurate and approximate calculations and the calculus. In 1875 the course was again lengthened to four years and the entrance requirements lessened, so that in mathematics algebra, to equations of the second degree, was the only requirement. In 1890 this was increased to algebra thru equations of the second

[4] The data for this institution and for the others to be considered are found in past catalogs and announcements.

degree and plane geometry. Since that time various changes in the entrance requirements in mathematics have been made until now they stand as given in Chapter II where will also be found the work in mathematics as scheduled for the college courses.[5]

(*ii*) *Lawrence Scientific School of Harvard University.*

In 1847 Abbott Lawrence gave $50,000 for the founding of a scientific school in connection with Harvard University. He desired "a school for the purpose of teaching the practical sciences—I have thought that these great branches to which a scientific education is to be applied amongst us should be first, engineering; second, mining in its extended sense, including metallurgy; third, the invention and manufacture of machinery." In honor of the donor this newly organized school was called the Lawrence Scientific School. The course in civil engineering was not begun until two years after the founding of the school. The course in mining was first offered in 1868, electrical engineering in 1888, mechanical engineering in 1894, and architecture in 1895. In 1907 the courses in the Lawrence Scientific School were made a part of the courses offered by Harvard College and Graduate School, thus making the course in engineering largely elective.

(*iii*) *Brown University.*

Brown University offered its first work in engineering in 1849 in the form of a partial course of two years called the "Engineering and Scientific Course." The work in mathematics included algebra, plane and solid geometry, mensuration, trigonometry, surveying and mechanics. The following year a subscription of $25,000 was raised and the University reorganized by Pres. Wayland, at which time the "New System" was adopted, which was in substance our full elective plan. One and one-half years' professional work in civil engineering was then offered which required as preparation about two full years of mathematics. At the same time a course in practical "chemistry applied to the arts" was also organized. In 1863 Brown University was made one of the beneficiaries of the government in accordance with the land grant act of 1862. Singularly enough no reference to work in engineering is found in the catalogs from 1863 to 1867. The catalog for 1867 offered a mechanical and agricultural course

[5] Chap. II, pp. 21-41.

which gave in the mechanical division geometry and algebra, the first year; trigonometry and engineering, the second year; besides several culture studies thruout the course. The final change in the mathematical schedule was made in 1901.

(*iv*) *Cumberland University.*

In 1852 Cumberland University added a School of Engineering to its other departments.

(*v*) *Sheffield Scientific School of Yale University.*

The Scientific School of Yale University dates from the establishment of its School of Applied Chemistry in 1847. The "Course in Engineering" of two years was first offered in 1853, and in 1864 was lengthened to three years with a variation along the lines of civil and mechanical engineering during the last two years. Mining engineering was added in 1866, electrical engineering in 1894 and sanitary engineering in 1900. The usefulness of the school was greatly enhanced by a liberal endowment in 1860 from Joseph E. Sheffield, in whose honor it took, two years later, the name of the Sheffield Scientific School. In 1864 it received congressional recognition and was made a participant in the aid given the "Land Grant Colleges." This revenue was taken from it by the Connecticut Legislature in 1892.

The chief feature in the mathematical schedule was the introduction in 1864 of the calculus of variations and in 1874 of "Elementary Theory of Numerical Approximations, Solutions of Higher Numerical Equations, Methods of Interpretations" for the second term of the first year. In 1880 the schedule in mathematics stood as follows: first year, analytic geometry and spherical trigonometry, one semester each; second year, elementary theory of functions, numerical equations and differential and integral calculus, one semester each. The following year geometry of three dimensions was put into the second year. In 1902 the first year was changed further to plane and solid analytic geometry, or an introduction to the calculus.

(*vi*) *University of Michigan.*

The University of Michigan holds the honor of being the first institution supported by a state to give an engineering course. It was also the only one of its kind before the passage of the Land

Grant Act in 1862. In 1853 its first course in civil engineering was offered in connection with the general science course. The work in mathematics was algebra, two terms, geometry, two terms, during the first year; trigonometry, one term, conics, two terms the second year; the calculus, one term the third year. During 1856-57 the two semester plan was put into effect and the mathematics schedule contained algebra and geometry the first half and geometry, trigonometry and mensuration the second half of the first year; descriptive and analytic geometry the first half and the calculus the second half of the second year. In the catalog for the year 1859 this statement is found: "The School of Engineering commences with the second year of the Scientific Course and is identical with it during the second and third years of that course." The additional years of technical work were then added which made algebra and geometry required subjects and placed trigonometry and analytic geometry in the first year of the engineering course, with the calculus in the second year, each of which was pursued for one-half year. A year later trigonometry was placed in the first year of the scientific course and hence required for entrance to the engineering course. The School of Mines was established in 1865 and made a separate institution in 1885. In 1865 the first three years of the Engineering and Scientific Courses were also made identical. This brought geometry (5-books), trigonometry and algebra to the first, analytic geometry and the calculus to the second year of the engineering course. Euclidean geometry was required for entrance after 1866, and plane trigonometry after 1890. In 1896 the School of Engineering was made a wholly separate department.

3) Military and Naval Institutions.

For the sake of completeness two institutions established during this period, the Virginia Military Institute and the United States Naval Academy, although not organized to give work in civil engineering will be considered briefly.

(*i*) *Virginia Military Institute.*

Virginia Military Institute, which was established as early as 1839, was patterned largely after the United States Military Academy. In 1860, thru bequests, it expanded along lines of general industrial education, only to have its plant destroyed by the northern

army. The cadets were transferred to Richmond, where the work was continued until the evacuation of that city in 1865. Since the reopening of the Institute in October, 1865, at Lexington, it has from time to time been enlarging its powers. The present courses for the first two and one-half years are the same, after which the student elects one of the following courses; chemical, electrical or civil engineering.

(*ii*) *United States Naval Academy.*

The establishment of a Naval Academy was first suggested by Hon. William Jones, Secretary of the Navy as early as 1814, but it was not until 1845 that the present school was created at Annapolis. Preceding this time the Navy had conducted a sort of midshipman apprentice course. As in all such courses, the instruction was largely practical, all of the theoretical matter being taken during the last six months of the course. The work prescribed in mathematics for this brief period was books 1, 2, 3, 4, and 6 of Playfair's Euclid, McClure's Spherics and Bourdon's Algebra. The Department of Mathematics in the new school was fully organized by 1850 and gave work in arithmetic and algebra the first year; geometry, trigonometry and descriptive geometry the second year; analytic geometry, the calculus and astronomy the third year; navigation and surveying the fourth year. From 1866 to 1870 little was done with analytic geometry. Arithmetic was omitted in 1871, at which time the calculus and mechanics were organized into one course.[6]

V. CONDITIONS IN ENGINEERING AT THE TRANSITION FROM THE SECOND TO THE THIRD PERIOD.

Toward the close of the second period conditions in engineering were changing at a rapid rate corresponding to external influences. By 1860 the consumption of iron had arisen to 919,370 tons. The growth and general development of the country was marvelous which resulted in many timesaving inventions.

[6] Park Benjamin, United States Naval Academy. New York, 1900. J. R. Soley, Rear-Admiral U. S. N., Historical Sketch of the United States Naval Academy. Government Printing Office, 1876.

1) Transportation.

The first iron truss bridge is thought to have been built by Trumbull in 1840 over the Erie Canal. During the ten years from 1850 to 1860 the railroad mileage had increased about 250%.[1] Two inventions closely related to the railroad industry were made at this time; the telegraph in 1844 and the air-brake in 1865.

2) Inventions.

The threshing machine was perfected about 1850 and the reaper ten years later. A sucessful fire engine was made in 1853. Great advances were made in dynamo machinery from 1860 to 1870. The two most notable events were the invention of cassions by M. Triger just before the middle of the century and the establishment of a plant for the manufacture of steel by the Bessemer process in 1859.

VI. THIRD PERIOD—1862 TO THE PRESENT.

The third period is that of the land grant colleges. They are the result of the "Morrill Land Grant Act" passed by Congress in 1862 in recognition of the need of technical education thruout the United States.

1) Morrill Land Grant Act and Subsequent Acts.

In substance the Morrill Land Grant Act is as follows: A grant of land was to be made to each state in the Union in the amount of 30,000 acres, or its equivalent, for each senator and representative in Congress to which the state was entitled by the apportionment of the census of 1860. The proceeds from the sale of these lands for each state were to form an endowment for the institutions established under the provisions of the act, and only the interest derived from the same to be available for the support of these institutions. It further required of such colleges that their "leading objects shall be, without excluding other scientific and classical studies, and including military tactics, to teach such branches of learning as are related to agriculture and the mechanical arts, in such a manner as

[1] See Table p. 4.

the legislatures of the states may respectively prescribe, in order to promote the liberal and practical education of the industrial classes in the several pursuits and professions in life." In 1892 Congress passed another bill known as the "Morrill Fund Act:" "to apply a portion of the proceeds of the public lands to the more complete endowment and support of the colleges for the benefit of agriculture and the mechanics arts established under the provisions of the act of 1862." According to this act the sum of $15,000 was to be paid each of the land grant colleges in 1890, and further, this was to be increased by $1,000 each year until 1900, after which the sum of $25,000 should be appropriated annually. March 4, 1907 Congress again increased this amount by what is known as the "Nelson Amendment." It provided that beginning July 1, 1908, the sum of $5,000 was to be added to each appropriation yearly until the amount was $50,000.[8]

2) Colleges of the Third Period.[9]

No review will be made of the colleges established during the third period, for their progress presents no radical changes, except at a very late date to which Chapter III will be devoted. A summary of the work offered by these various institutions will be found in the next chapter.[10]

3) Growth of the Engineering Colleges.

The graph giving the increase in the number of engineering colleges serves best to show the marvelous growth in engineering education since the recognition of its need by the United States Government.[11] The following figures regarding the increase in students taking professional courses from 1878 to 1900 may be of interest:

theology	8,079	or 87%
medicine	26,088	or 142%
law	11,835	or 294%
engineering	9,659	or 516%[12]

[8] I. E. Clark, A.M., Education in the Industrial and Fine Arts in the United States. Vol. IV, p. 838. U. S. Bureau of Education, 1898.

[9] For list of Land Grant Colleges see foot-note p. 21.

[10] Chap. II, pp. 21-41.

[11] Chap. II, p. 42.

[12] I. O. Baker, Presidential Address at the Annual Meeting of the Society for the Promotion of Engineering Education, 1900. Proceedings vol. VIII, p. 11.

VII. SKETCH OF CHANGES IN MATHEMATICS IN ENGINEERING COLLEGES.

The engineering courses offered by all of the institutions existing during the second period, with the exception of Rensselaer Polytechnic Institute, were only continuations of their previously existing literary or general science courses in which engineering subjects had been introduced into the work of the last two, or at most, three years. For this reason the work in mathematics for engineers really began with that given in the general science course in those institutions where the engineering course was a continuation of the former.

The second period may be regarded as the formative and experimental stage in the development of the engineering colleges; changes in the mathematical work were numerous, sometimes increasing often diminishing the amount required for entrance with corresponding shifting of the collegiate curriculum. During the third period changes in entrance requirements have always been in the nature of increases. During this third period there have arisen institutions with widely differing purposes, presenting a wide range of variation both in the work required for entrance as well as that offered in the curriculum, and showing corresponding irregularity in the development of the mathematical courses. An exact resume of the changes is, of course, impossible. The following sketch is based upon the changes in twenty of the leading institutions and is quite characteristic of the general development.

1) Changes in Entrance Requirements.

About 1850 some algebra began to be required for entrance, by 1870 several of the leading institutions required one year and by 1908 85% have made a requirement of one and one-half years of algebra.[13] As early as 1855 a very few institutions required plane geometry for entrance, and by 1875 there were still very few requiring it, while it was quite generally required in 1880. The University of Michigan and Stevens Institute were the first to require trigonometry, which they did in 1890. Since then more and more have made it a required subject until now 18% do so.[13] A few institutions

[13] Table of entrance requirements Chap. II, p. 43.

required solid geometry in the early eighties and between 1885 and 1900 a majority of the institutions made this a requirement, since which time there has been little change up to the present.[13]

2) Collegiate Curriculum.

As more work was required for entrance a proportionate amount of a more advanced nature was added to the curriculum. The total time given to mathematics was at first about one year but has been increased by easy stages until now about two full years are given to it. Trigonometry was either required for entrance or formed a part of the college work in every course organized. Analytic geometry has, to an extent, also been a part of nearly every course. The time devoted to it has been more than doubled and its position in the curriculum changed from the last part of the second year or first part of the third year to the first year.[14] The calculus was made a part of some engineering schedules as early as 1855 but it came late in the course, was taken only for one-third or one-half of a year and did not enter into the work of the course to any extent. The increase in the attention devoted to the calculus has been more gradual than in any other subject, save that of algebra. The time has been greatly increased, a whole year now being given to it by most institutions, and it has been placed earlier in the curriculum; namely, in the second year.

3) General Trend.

The general trend of late years has been to require one and one-half years of algebra, plane and solid geometry for entrance:[15] and to complete algebra, trigonometry and analytic geometry in the first year, with the calculus the second year.[16] The ratio of the time given to each subject will be found in Chapter II.

[13] Table of entrance requirements. Chap. II, p. 43.
[14] Table of collegiate subjects. Chap. II, p. 44.
[15] Table of entrance requirements. Chap. II, p. 43.
[16] Table of collegiate mathematics. Chap. II, p. 44.

VIII. PROGRESS OF INSTRUCTION IN MATHEMATICS.

It is not our purpose here to review the progress of mathematical instruction in the United States.[17] During the early years of the engineering colleges there was a great deal of poor teaching, owing to the fact that instructors were not especially prepared in mathematics. The spirit of scientific instruction, the learning to do by doing, which has been adopted extensively by the professional departments has not received nearly so much recognition by the instructors of mathematics. But little has been done along the line of formulating and presenting the work in courses especially adapted and correlated to the professional subjects, and that little only recently.[18] So far slight attention has been paid to the training of instructors as teachers.

IX. SUMMARY.

The apprenticeship system and the Military Academy furnished all of the engineering education until about 1835. With the founding of the Rensselaer School in 1824 began the formative period of the engineering colleges which lasted until about the time of the Civil War. The courses were not highly specialized and were mostly variations for the last two years of the existing courses in the literary institutions. There was no stability in the work in mathematics, either as to time allotted, courses offered or their position in the schedule. Although the National Government had quite early made provision for military and naval instruction it was not until 1862 that it recognized the need of technical instruction along civil lines, by the passage of the Land Grant Act. Since that time institutions of various grades have been established. Contrasted with the changes of the formative period those of the present have been continuously progressive; increasing the quantity as well as the quality of the work in mathematics, placing the mathematical courses earlier in the curriculum so as to make them precede most of the technical work, with the gradual evolution of the present schedule, while close correlation of the work in mathematics with that in the technical subjects has but lately been taken up.

[17] For a comprehensive discussion of this phase see Florian Cajori, The Teaching and History of Mathematics in the United States, Government Printing Office, 1890, pp. 98-106.

[18] Chap. III.

CHAPTER II.

ENGINEERING COLLEGES IN THE UNITED STATES IN 1902.

I. CONTENTS.

The present chapter presents in condensed form data bearing on the mathematical instruction of freshmen engineers in the various institutions in the United States offering one or more courses in engineering. It includes all institutions granting a degree for work in engineering and graduating a class of at least six from such courses, as compiled from the list contained in the Report of the Commissioner of Education for 1908. The names of the institutions are arranged by states in alphabetical order.

II. SOURCES OF DATA.

The information for the following table has been obtained largely from the current catalogs of the various institutions. When this information was lacking or given in such a manner as to require considerable interpretation, a letter was sent to the institution in question, asking for the missing data or for corrections of the interpretations made. If no corrections were received the conclusions were assumed to be correct and recorded as such in the table. In a few instances it has been impossible to obtain the necessary data thru either catalogs or letters, and hence parts of the table have necessarily been left blank. While every effort has been made to guard against errors in making this table, it is too much to expect that some have not crept in. It is only hoped that the errors are not grave ones and that no institution has been seriously misrepresented.

III. EXPLANATIONS OF TABLES.

The following explanations are necessary to an understanding of the tables on pages 21 to 41.

a) *Column* 1 is self explanatory.

b) *Column* 2 gives the date at which the first engineering courses of the various institutions were established.

c) *Column* 3 gives a list of the courses, as, Civil Engineering, Electrical Engineering, etc., offered by these institutions at the present time. The following abbreviations are used:

C. E., Civil Engineering.
Mi., Mining Engineering.
Ch., Chemical Engineering.
Sani., Sanitary Engineering.
A. E., Architectural Engineering.
A., Architecture.
E. E., Electrical Engineering.
M. E., Mechanical Engineering.
Irrig., Irrigation Engineering.
Mun., Municipal Engineering.
N. A., Naval Architecture.
Met., Metallurgy.

d) *Column* 4 contains the entrance requirements in mathematics. The statements found in the various catalogs have occasionally had to be modified before being recorded here, since in some instances neither the time devoted to the subject nor the ground covered was definitely stated and because some fixed system of units is essential for comparison. The usual high school unit, which requires at least four recitations per week of forty-five minutes each for thirty-six weeks, was the one selected with the following time limit: algebra to quadratics, one year; elementary algebra completed, one and one-half year; plane geometry, one year; solid geometry, one half year; and trigonometry, one-half year. This system of counting conforms quite closely to the recommendation of the Committee on Entrance Requirements given before the Society for the Promotion of Engi-

neering Education in 1901,[1] and 1902.[2] If no definite entrance requirements were stated either as to time or as to work, deductions had to be drawn from the statement of the courses offered in mathematics for the freshman year. Wherever elective courses in entrance mathematics are found the minimum requirement is given and the elective courses placed in the freshman year of the college work.

e) *Column* 5 states for which course (Civil Engineering, Mechanical Engineering, or the like) the data of the succeeding seven columns are tabulated. This was found necessary as no one course was given by all of the institutions. To preserve as much uniformity as possible, the Mechanical Engineering course, whenever given, has been scheduled.

f) *Column* 6 contains, in condensed form, the mathematical work of the curriculum. This is all required except that (as mentioned above) a few of the institutions will accept for entrance some of the courses scheduled under the freshman year. For the sake of uniformity the time devoted to each subject has been reduced from term, semester or year hours to that of the actual number of recitation hours; again for the sake of uniformity and also to simplify the work very considerably, thirty-six weeks have been selected as the length of the school year.

g) *Columns 7 to* 12 inclusive contain the number of hours given

[1] "a" Elementary Algebra.

1) To quadratics—The four fundamental operations for rational algebraic expressions, factoring, highest common factor, lowest common multiple, complex fractions, equations of the first degree of one or more unknowns, radicals, fractional exponents.

2) Quadratics and beyond—Quadratics in one or more unknowns, ratio and proportion, progressions, elements of permutations and combinations, Binomial Theorem with integral exponents and the use of logarithms.

"b" Advanced Algebra.

To include the elementary treatment of infinite series, undetermined coefficients, Binomial Theorem for fractional exponents, theory of logarithms, determinants and the elements of the theory of equations.

Plane Geometry.

To include original exercises and numerical problems.

Solid Geometry.

To include properties of straight lines, planes, dyhedral and polyhedral angles, polyhedrons, inclined prisms, pyramids, regular solids, cylinders, cones and spheres, spherical triangles and measurement of surfaces and solids.

Comittee on Entrance Requirements, Proceedings of the Society for the Promotion of Engineering Education, vol. IX, p. 267.

[2] The next year it reported the following change desirable: that permutations and logarithms be omitted; that imaginaries should be emphasized. Ibid., vol. X, p. 200.

The part marked "Elementary" under algebra is that for the one and one-half years' high school work, with the exception of the use of logarithms which are usually taught with trigonometry. The "Advanced" is the usual first year's college work.

to mathematics (M), professional subjects (P), semi-professional subjects (Sp), language (L), miscellaneous subjects (Mi), and the total (T). The hours are computed as for column six. The number in column seven is merely the sum of the number of hours recorded for the various subjects in the one immediately preceding it. In some catalogs surveying and mechanics are placed under the Department of Mathematics. Whenever it has been possible to differeniate these from the purely mathematical subjects they are counted as professional. This has been done to preserve uniformity, as in most of the institutions these subjects are taught by the professional departments. Semi-professional subjects include physics, geology, chemistry, astronomy and mineralogy. Some of these become professional work when a course other than Mechanical Engineering has been scheduled. The hours given under language include English.

The general rule is to give few electives except where the work in engineering is a modification of a literary course. Such electives are scheduled under miscellaneous subjects. Unless otherwise mentioned the time required to complete these courses is four academic years.

h) *Column* 13 gives the percentages of the total time which each institution devotes to mathematics.

‡IV. Tables of Data, Engineering Colleges.

Institutions.	Date	Courses Given.	Entrance Requirements.	Course Scheduled.	Mathematical Course.	M.	P.	Sp.	L.	Mi.	T.	%M.
* Alabama Polytech. Inst. ..... Auburn, Ala.	1872	C.E., E.E., M.E., Mi., Ch., A.	Arith. Alg. .....1† P. Geom..... ½	M.E.		648	1344	468	420	348	3228	20.1
University of Alabama..... Tuscola, Ala.	1880	C.E., Mi., E.E., M.E.	Alg. ..... 2 P. Geom..... 1	M.E.	F.—P. Trig..... 72 F.—P. Anal. ..... 72 S.—Calc. 144	288	1926	468	216	78	2976	9.7
* University of Arizona..... Tuscon, Ariz.	1891	Mi., C.E., M.E., Met.	Alg. ..... 2 P. & S. Geom. 1½ Trig. ..... ½	M.E.	F.—Alg. with 12 hrs lab. periods.... 90 F.—Anal. ditto ..... 90 S.—Calc. 108	288	1314	414	108	180	2304	12.5
* University of Arkansas..... Fayetteville, Ark.	1872	M.E., E.E., C.E., Mi., Met.	Alg. ..... 1 P. Geom..... 1	M.E.	F.—Alg. ..... 54 F.—S. Geom. ..... 54 F.—Trig. ..... 54 F.—Anal. ..... 54 S.—Anal. & Trig.....108 S.—Calc. ..... 72 J.—Alg. ..... 24 J.—Calc. 48	468	1548	216	108	0	2340	20.0
[1] Leland Stanford Jr. Univ..... Stanford, Cal.	1891	C.E., M.E., E.E.	Alg. ..... 1½ P. & S. Geom. 1½ P. Trig. .... ½	M.E.	F.—Alg., Trig., Anal. 180 S.—Calc. 108	288	1476	396	0	0	2160	13.3

* Land Grant Colleges.
[1] Course is only suggestive; not prescribed.
† Refers to years of academic study.
‡ For explanations of the tables see pp. 18-20.

Institutions.	Date	Courses Given.	Entrance Requirements.	Course Scheduled.	Mathematical Course.	M.	P.	Sp.	L.	Mi.	T.	%M.
Throop Polytech. Inst. ..... Pasadena, Cal.	1891	M.E., E.E.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—P. Trig. ..... 72 F.—P. & S. Anal.... 72 S.—Calc. 108	256	1298	252	108	270	2180	11.6
[2] St. Mary's College..... Oakland, Cal.	——	C.E., Mi.	Alg. ..... 1½ P. Geom..... 1	C.E.	F.—Alg. .....108 F.—S. Geom. .....108 F.—Trig. .....108 S.—Anal. .....108 S.—Calc. 108	540	216	324	432	108	1620	33.3
*[3] University of California..... Berkeley, Cal.	1869	M.E., C.E., Mi., A.	Alg. ..... 1½ P. & S. Geom. 1½ P. Trig. .... ½	M.E.	F.—Courses for Engineers in Alg., Anal., El. Calc. 108 S.—Calc. 108	216	1332	504	0	594	2646	8.1
Univ. of Southern Cal..... Los Angeles, Cal.	1907	E.E., C.E.	P. & S. Geom. 1½ Alg. 2	E.E.	F.—Alg. & P. Trig... 90 F.—P. Anal. ..... 90 S.—Calc. 126	306	1542	414	288	0	2550	12.0
* State Agri. College ..... Ft. Collins, Colo.	1879	M.E., C.E., E.E., Irrig.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Adv. Alg. ..... 60 F.—P. & S. Trig..... 60 F.—P. & S. Anal..... 60 S.—Calc. 120	300	1652	300	384	108	2744	10.9
[4] Colorado College ..... Colorado Springs, Colo.	1903	E.E., C.E., Mi., Irrig.	Alg. ..... 1½ P. & S. Geom. 1½	E.E.	F.—Alg. ..... 72 F.—P. Trig. ..... 72 S.—Anal. ..... 90 S.—Calc. 126	360	1476	396	288	0	2520	14.3
Colo. School of Mines ..... Golden, Colo.	1874	Mi., Met.	Alg. ..... 1½ P. & S. Geom. 1½	Mi.	F.—Alg. ..... 90 F.—P. & S. Trig..... 54 F.—P. & S. Anal..... 90 F.—Ele. Analysis ... 54 S.—Calc. 144	432	2277	781	36	0	3526	12.2

[2] Data approximate only.
[3] Summer work included in schedule of hours.
[4] Summer memoirs on practical work required.

University of Colorado ..... 1893 Boulder, Colo.	C.E., E.E., M.E., Ch.	Alg. ..... 1½ M.E. P. & S. Geom. 1½	F.—Alg. ..... 54 F.—Trig. ..... 36 F.—Anal. ..... 90 S.—Calc. 180	360	1566	360	180	54	2520	14.1
Trinity College ..... 1905 Hartford, Conn.	C.E., E.E., Sani.	Alg. ..... 1 C.E. P. Geom..... 1	F.—S. Geom., Alg., P. Trig. .....108 S.—S. Trig., Anal....108 J.—Calc. 108	324	756	216	216	648	2160	15.0
[5] Sheffield Scientific School..... 1853 of Yale Univ. New Haven, Conn.	C.E., M.E., E.E., Mun., Sani., Prep. to Mi.	Alg. ..... 2 M.E. P. & S. Geom. 1½ P. Trig. .... ½	F.—P. Anal. ..... 81 F.—S. Anal. ..... 27 S.—Calc. 135	243	648	270	540	0	1701	14.3
* Delaware College ..... 1870 Newark, Del.	M.E., C.E., E.E.	Alg. ..... 1½ M.E. P. Geom..... 1	F.—P. & S. Trig., Surveying .....90 F.—S. Geom., Alg., Surveying ..... 90 S.—P. & S. Anal., H.P.C. ..... 72 S.—Calc. ..... 72 J.—App. Calc. & Dif. Equas. 108	432	1050	252	432	144	2310	14.3
Catholic Univ. of America.... —— Washington, D. C.										
George Washington Univ. ..... 1884 Washington, D. C.	C.E., M.E., E.E., A	Alg. ..... 1½ M.E. P. & S. Geom. 1½ P. Trig. .... ½	F.—P. & S. Trig..... 33 F.—P. & S. Anal..... 75 S.—Calc. ..... 81 S.—Dif. Eqs. 27	216	1566	396	324	0	2502	8.6
John B. Stetson, Univ. ..... —— De Land, Fla.	C.E., M.E., E.C. Ch.	Alg. ..... 1½ M.E. P. & S. Geom. 1½	F.—Trig. ..... 60 F.—Alg. ..... 30 F.—Anal. ..... 90 S.—Calc. 180	360	870	600	120	210	2160	16.6

[5] Courses three years long. Two years post-graduate work is also given.

Institutions.	Date	Courses Given.	Entrance Requirements.	Course Scheduled.	Mathematical Course.	M.	P.	Sp.	L.	Mi.	T.	%M.
* University of Florida ..... Gainsville, Fla.	1905	General.	Arith. .... Alg. ..... 1½ P. Geom..... 1	Gen.	F.—S. Geom., P. Trig. 180 S.—Alg., Anal. ..... 108 S.—Calc. ..... 108 J.—Calc. ..... 108 J.—Eqs., & Dif. Eqs.. 108	612	1260	396	396	144	2808	21.8
Georgia School of Technology Atlanta, Ga.	1888	C.E., M.E., E.E. Ch., Textile.	Alg. ..... 1½ P. Geom..... 1	M.E.	F.—S. Geom. ..... 36 F.—Alg. ..... 54 F.—P. Trig. ..... 54 F.—Anal. ..... 36 S.—Anal. ..... 54 S.—Calc. ..... 126 J.—Dif. Eqs. & Eqs.. 54	414	1134	504	576	0	2628	15.7
* North Ga. Agri. College..... Dahlonega, Ga.	——											
University of Georgia ..... Athens, Ga.	1872	C.E., E.E., M.E.	Arith. .... Alg. ..... 1½ P. Geom..... 1	M.E.	F.—S.Geom., P. Trig. 180 S.—Anal. ..... 108 S.—Calc. ..... 108 J.—Calc. ..... 108 J.—Dif. Eqs. & Eqs.. 108	612	1512	396	396	144	3060	20.0
* University of Idaho ..... Moscow, Idaho.	1892	M.E., E.E., C.E., Mi.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Alg. ..... 90 F.—P. & S. Trig..... 54 S.—Anal. & H.P.C... 72 S.—Calc. ..... 72 J.—Calc. 72	360	1728	288	216	108	2700	13.3
Armour Inst. of Technology... Chicago, Ill.	1892	M.E., E.E., C.E., Ch., Fire Protection.	Alg. ..... 1½ P. & S. Geom. 1½ P. Trig. .... ½	M.E.	F.—Alg.(Practical) . 90 F.—Anal. & Calc..... 90 S.—Calc. 90	270	2034	432	108	216	3060	9.0
James Millikin University ..... Decatur, Ill.	1901	C.E., M.E., E.E.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Alg. for Engin'rs 54 F.—P. & S. Trig. " 36 F.—P. & S. Anal. " 90 S.—Calc. & Dif. Eqs. for Engineers. 144	324	1548	324	108	144	2448	13.2

*Northwestern University ..... 1907 Evanston, Ill.	General.	Alg. ..... 1½ Gen. P. & S. Geom. 1½	F.—Alg., P. Trig..... 90 F.—P. & S. Anal..... 90 S.—Calc. .....108 J.—Calc. & Dif. Eqs. 108	396	432	648	324	0	1800	22.0
*University of Illinois ..... 1868 Urbana, Ill.	C.E., A., A.E., E.E., M.E., Sani. & Munic., R.R. in M.E., in C.E., in E.E.	Alg. ..... 1½ M.E. P. & S. Geom. 1½	F.—Alg. ..... 54 F.—Trig. .....36 F.—Anal. ..... 90 S.—Calc. 144	324	1584	162	252	216	2538	12.8
*Purdue University .....1874 Lafayette, Ind.	C.E., M.E., E.E.	Alg. ..... 1½ M.E. P. & S. Geom. 1½	F.—Alg. ..... 90 F.—Trig. ..... 90 S.—Anal. ..... 72 S.—Calc. ..... 72 J. Calc. 72	396	1577	306	342	194	2815	14.2
Rose Polytechnic Institute .....1883 Terre Haute, Ind.	C.E., M.E., E.E., A., Ch.	Alg. ..... 1½ M.E. P. & S. Geom. 1½	F.—Alg. ..... 72 F.—Anal. Trig. ..... 54 F.—Calc. & Anal., (Graphs, etc.). 36 S.—Calc. & applications to P.Anal. 144	306	1350	378	234	342	2610	11.7
University of Notre Dame..... 1874 Notre Dame, Ind.	M.E., E.E., C.E.	Alg. ..... 1½ P. & S. Geom. 1½ Trig. ..... ½ M.E.	F.—Alg. ..... 90 F.—P. & S. Anal. & H.P.C. ..... 90 S.—Calc. .....150 S.—Dif. Eqs. 30	360	1602	486	288	0	2736	13.2
Cornell College ..... 1873 Mt. Vernon, Ia.	C.E.	Alg. ..... 1½ C.E. P. & S. Geom. 1½	F.—Alg. ..... 40 F.—P. Trig. ..... 50 F.—P. & S. Anal..... 90 S.—Alg. (Adv.)..... 45 S.—Calc. 135	360	720	180	144	828	2232	16.1

*Only three of four years scheduled. Six weeks summer work required yearly in practice shop.

Institutions.	Date	Courses Given.	Entrance Requirements.	Course Scheduled.	Mathematical Course.	M.	P.	Sp.	L.	Mi.	T.	%M.
* Iowa State A. & M. College.. Ames, Ia.	1868	C.E., M.E., E.E., Mi.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Alg. ..... 72 F.—Trig. ..... 54 S.—P. Anal. ..... 54 S.—Anal. ..... 36 S.—Calc. 126	342	1476	468	252	90	2628	13.0
University of Iowa ..... Iowa City, Ia.	1869	C.E., M.E., E.E., Ch., Mi., Sani.	P. & S. Geom. 1½ Alg. 1½	M.E.	F.—Alg. ..... 45 F.—Trig. ..... 45 F.—P. & S. Anal.... 90 S.—Calc. .....135 S.—Theory Equas.... 45	360	1512	360	180	117	2529	14.1
* Kansas State A. & M. College Manhattan, Kan.	1897	M.E., E.E., C.E., A.	Alg. ..... 1½ Arith.	M.E.	F.—Geom. .....120 F.—Trig. ..... 60 S.—Alg. ..... 60 S.—Anal. ..... 60 S.—Calc. ..... 60 J.—Calc. 60	420	1148	600	240	468	2876	14.6
University of Kansas ..... Lawrence, Kan.	1891	C.E., M.E., E.E., Ch., Mi.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Alg. ..... 54 F.—P. Trig. ..... 36 F.—Anal. ..... 36 F.—Calc. ..... 54 S.—Anal. ..... 36 S.—Calc. 90	306	1701	360	324	0	2691	11.4
* State University of Kentucky Lexington, Ky.	1887	C.E., M.E., E.E., Mi.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Alg. .....100 F.—P. Trig. ..... 75 S.—P. Anal.....125 S.—S. Anal. & H.P.C. 50 S.—Calc. .....100 J.—Calc. 75	525	2764	387	105	287	3436	15.2

* Louisiana State University... Baton Rouge, La.	1874	C.E., M.E., E.E., Ch.	Alg. ..... 2 P. Geom..... 1	M.E.	F.—Alg. .....108 F.—S. Geom. ..... 54 F.—Trig. ..... 54 S.—Alg. ..... 54 S.—Anal. ..... 54 S.—Calc. ..... 54 J. —Calc. 108	486	1224	390	324	72	2496	19.4
Tulane University ..... New Orleans, La.	1890	M.E., C.E., Ch., A., A.E., Sugar.	Alg. ..... 1½ P. Geom..... 1	M.E.	F.—S. Geom., Alg.... 72 F.—P. & S. Trig..... 72 S.—P. & S. Anal..... 96 S.—Calc. ..... 48 J.—Calc. 60	348	958	486	540	180	2502	13.9
* University of Maine ..... Orono, Me.	1868	C.E., E.E., M.E., Ch.	Alg. ..... 2 P. & S. Geom. 1½	M.E.	F.—Adv. Alg. ..... 70 F.—P. Trig. ..... 60 F.—Anal. ..... 50 S.—Conics & Anal... 40 S.—Calc. 140	360	1320	342	360	294	2676	13.4
* Maryland Agri. College ..... College Park, Md.	1894	M.E., E.E., C.E.	Arith. .... 1 Alg. 1	M.E.	F.—Rev. Ele. Math... 36 F.—Alg. .....192 F.—P. Geom..... 60 S.—P. Geom. ..... 60 S.—S. Geom..... 60 S.—P. Trig..... 60 J. —Anal. ..... 60 J. Calc. 120	684	1224	424	732	412	3440	18.8
St. John's College ..... Annapolis, Md.	1885	General.	Arith. ..... ½ Alg. ..... 2 P. & S. Geom. 1½	Gen.	F.—P. & S. Trig., and Anal. .....144 S.—Calc. 144	288	1134	252	396	450	2520	11.5
† Lawrence Scientific School.... of Harvard Univ. Cambridge, Mass.	1850	C.E., M.E., E.E.	Alg. ..... 1 P. & S. Geom. 1½	M.E.	F.—Alg. 54; Trig. 54; Anal. 54, or same with lab..144 S.—Calc. 216	360 or 369	2112	384	216	0	3072 or 3081	12.0

Institutions.	Date	Courses Given.	Entrance Requirements.	Course Scheduled.	Mathematical Course.	M.	P.	Sp.	L.	Mi.	T.	%M.
*[8] Mass. Agricultural College.. Amherst, Mass.	1867	C.E.	Alg. ..... 1½ P. Geom..... 1	C.E.	F.—Alg. ..... 90 F.—P. & S. Trig..... 54 F.—S. Geom..... 36 S.—P. Anal. & H.P.C.. 72 S.—Calc. 72	324	468	252	486	1008	2538	12.8
*[9] Mass. Inst. of Technology.. Boston, Mass.	1865	C.E., M.E., Mi., Met., A., E.E., Ch., Sani., N.A.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—P. Trig. ..... 30 F.—Spcl. Course in Anal. & Calc... 90 S.—Calc. ..... 45 S.—S. Anal. & Calc.. 45 J.—Dif. Equas..... 30	240	909	227	164	138	1678	14.3
Tufts College ..... Tutfs Col. Sta., Boston, Mass.	1869	C.E., M.E., E.E., Ch.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Alg. ..... 54 F.—Trig. ..... 54 F.—P. Anal. ..... 54 S.—Calc. .....108 J.—Dif. Equas..... 36	306	1476	354	342	108	2586	11.8
[10] Worcester Polytechnic Inst... Worcester, Mass.	1868	M.E., C.E., Sani., E.E., Ch.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Alg. ..... 72 F.—Trig. ..... 54 S.—Anal. & Calc..... 90 S.—Calc. 90	306	1476	354	342	108	2586	11.8
* Mich. State Agri. College.... East Lansing, Mich.	1885	M.E.	Alg. ..... 1½ P. Geom..... 1	M.E.	F.—Alg. .....120 F.—S. Geom..... 60 F.—P. & S. Trig..... 60 S.—P. & S. Anal., H.P.C. ..... 60 S.—Calc. 120	420	1986	348	240	672	3666	11.5

[7] Course scheduled is for "Master's degree in Mechanical Engineering."
[8] Course is highly elective.
[9] The courses mentioned are subdivided through specialization as Hydraulic, Railroad under C.E. The hours are 1/3 of the sum of time allotted to preparation and to the exercises to follow general scheme adopted.
[10] Three summers of shop work a total of 9 weeks, is included under professional hours.

[11] Mich. College of Mines..... Houghton, Mich.	1886	Mi.	Alg. ..... 1½ P. & S. Geom. 1½	Mi.	F.—Alg. .....100 F.—P. Trig..... 40 F.—S. Trig. ..... 30 F.—P. & S. Anal..... 90 S.—Calc. 110	370	2130	0	0	0	2500	14.8
[12] University of Michigan..... Ann Arbor, Mich.	1853	C.E., M.E., E.E., Ch., N.A., A., A.E., Geol.	Alg. ..... 1½ P. & S. Geom. 1½ P. Trig. .... ½	M.E.	F.—Alg. ..... 36 F.—P. Anal..... 72 F.—S. Anal. ..... 36 S.—Calc. .....126 J. —Dif. Eqs. 18	288	1440	324	432	36	2520	11.5
* University of Minnesota..... Minneaoplis, Minn.	1872	C.E., M.E., E.E.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Alg. & Anal. Trig. 90 F.—Trig. & Anal..... 90 S.—P. & S. Anal..... 72 S.—Calc. ..... 72 J. Calc. & Mechs..... 108	432	1404	504	360	216	2916	14.8
* Miss. A. & M. College..... Agricultural College, Miss.	1890	M.E., E.E., C.E., Mi., Textile.	Arith.	M.E.	F.—Ele. Alg. .....180 F.—P. Geom. ..... 60 S.—S. Geom. .....120 S.—Alg. ..... 60 J. —P. & S. Trig..... 60 J. —P. Anal. ..... 60 J. Calc. ..... 60 Sr.—Calc. 60	660	1208	512	552	552	3484	18.8
University of Mississippi..... University, Miss.	1900	C.E., Muni., Sani., E.E.	Alg. ..... 1 P. Geom..... 1	E.E.	F.—Alg. ..... 60 F.—S. Geom. ..... 60 F.—P. & S. Trig..... 60 S.—P. Anal..... 72 S.—Calc. ..... 36 J. —Calc. 108	396	1152	216	360	216	2340	16.9

[11] Professional and semi-professional had to be taken together.

[12] Gives also 6 year courses in each in connection with Literary College. Mechanics is taken in connection with the calculus and differential equations in second and third years. One summer of 8 weeks is included.

Institutions.	Date	Courses Given.	Entrance Requirements.	Course Scheduled.	Mathematical Course.	M.	P.	Sp.	L.	Mi.	T.	%M.
Washington University ..... St. Louis, Mo.	1870	C.E., M.E., E.E., Ch., A., A.E.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Alg. ..... 90 F.—P. Trig. ..... 36 F.—Anal. ..... 54 S.—Calc. .....126 J.—Dif. Eqs. 54	360	1674	468	270	0	2772	13.0
Christian Brothers College..... St. Louis, Mo.	——											
*University of Missouri..... Columbia, Mo.	1871	C.E., E.E., M.E., Ch., Sani., Mi., Met.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Alg. ..... 36 F.—P. & S. Trig..... 54 F.—P. & S. Anal..... 90 S.—Calc. 180	360	1512	360	108	36	2376	15.1
*Mont. State Coll. of Agri. and Mechanics Arts ..... Bozeman, Mont.	1893	C.E., E.E., M.E.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Alg. ..... 54 F.—P. Trig. ..... 36 F.—P. & S. Anal..... 72 S.—Calc. 180	342	1638	396	144	252	2772	12.3
[19] University of Montana..... Missoula, Mont.	1895	General.	Alg. ..... 1½ P. & S. Geom. 1½	Gen.	F.—Math. for Engrs. 180 S.—Continuation ....144 J. Dif. Eqs. ..... 54 J.—Least Sqs. 36	414	1512	288	144	0	2358	16.2
* University of Nebraska..... Lincoln, Neb.	1880	C.E., E.E., M.E.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—P. Trig. & Anal.. 90 F.—Anal. & Calc..... 90 S.—Anal. & Calc.....180	360	1368	288	72	198	2286	15.7
* Nevada State University..... Reno, Nev.	1886	Mi., M.E., C.E.	Alg. ..... 1½ P. & S. Geom. 2	M.E.	F.—Alg. .....36 F.—S. Geom. ..... 18 F.—P. Trig. ..... 36 F.—P. & S. Anal..... 90 S.—Calc. 180	360	1728	450	180	306	3024	11.9

[19] Also gives graduate courses of two years in M.E., C.E., and E.E.

* New Hampshire College..... Durham, N. H.	1866	M.E., E.E., Ch.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Alg. ..... 72 F.—P. & S. Trig..... 45 S.—Anal. ..... 90 S.—Calc. 90	297	1692	342	324	270	2925	10.2
[14] Thayer School of Civil Eng.. of Dartmouth College. Hanover, N. H.	1871	C.E.	Thru Calc.	C.E.	None.	...	....	...	...	...	....	....
[15] John C. Green School of Sci. Princeton University. Princeton, N. J.	1873	C.E.	Alg. ..... 1½ P. & S. Geom. 1½ P. Trig. .... ½	C.E.	F.—Alg. ..... 72 F.—P. & S. Trig..... 36 F.—Conics ..... 36 S.—Conics and Calc. 108	252	1566	486	180	0	2484	10.1
* Rutgers College ..... New Brunswick, N. J.	1864	M.E., E.E., C.E., Ch.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Col. Alg. ..... 60 F.—Trig. ..... 60 F.—Anal. ..... 60 S.—Anal. ..... 60 S.—Calc. .....120 J. Calc. 60	420	1146	216	432	1378	3592	11.7
Stevens Institute ..... Hoboken, N. J.	1871	M.E.	Alg. ..... 1½ P. & S. Geom. 1½ P. Trig. .... ½	M.E.	F.—Logs, Dets., Eqs., P. Anal. ..... 60 F.—H.P.C. & Calc... 60 S.—Calc. ..... 60 S.—Application Calc.. 60	240	1536	314	430	0	2520	9.6
* New Mex. Coll. of Agri. and Mechanics Arts ..... Messila Park, N. M.	1890	C.E., E.E., M.E.	Arith. .... Alg. ..... 1½ P. Geom..... 1	M.E.	F.—S. Geom. ..... 54 F.—Alg. ..... 54 F.—Trig. ..... 72 S.—P. Anal. ..... 54 S.—Calc. ..... 54 J.—Calc. 54	342	1422	486	432	198	2880	11.8
New Mex. School of Mines.... Socorro, N. M.	1895	Mi., Met., C.E.	Alg. ..... 1½ P. & S. Geom. 1½	Mi.	F.—Alg. .....144 F.—P. & S. Trig..... 72 F.—P. & S. Anal..... 72 S.—Calc. 144	432	2289	582	180	0	3483	17.0

[14] Two full years of professional work given.
[15] Two additional years graduate work in C.E., and E.E. Two weeks summer field work included in hours.

Institutions.	Date	Courses Given.	Entrance Requirements	Course Scheduled.	Mathematical Course.	M.	P.	Sp.	L.	Mi.	T.	%M.
University of New Mexico.... Albuquerque, N. M.	1906	M.E., E.E., C.E., Mi.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Col. Alg. ..... 54 F.—P. & S. Trig..... 36 F.—P. Anal. ..... 36 S.—P. & S. Anal., H.P.C. ..... 36 S.—Calc. ..... 54 J.—Calc. 54	270	720	432	216	882	2520	10.7
Polytechnic Inst. of Brooklyn.. Brooklyn, N. Y.	1870	M.E., E.E., C.E., Ch.	Alg. ..... 1½ P. & S. Geom. 1½ Trig. ..... ½	M.E.	F.—Course in Mod. Math. for Eng. 144 S.—Calc. 108	252	1514	468	378	180	2792	9.0
Clarkson Polytechnic Institute.. Potsdam, N. Y.	1896	M.E., C.E., E.E., Ch.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—P. & S. Trig..... 72 F.—P. Anal. ..... 72 S.—Calc. ..... 90 S.—Dif. Eqs. 45	279	1485	360	252	180	2556	10.9
Coll. of the City of New York New York, N. Y.	1890	M.E.	Alg. ..... 3 P. Geom..... 1 P. Trig. .... ½	M.E.	F.—P. & S. Anal.....144 S.—Calc. 144	288	288	324	288	1440	2628	10.9
[16] Columbia University ..... New York, N. Y.	1863	Mi., C.E., M.E., E.E., Ch., Met.	Alg. ..... 2 P. & S. Geom. 1½ P. Trig. .... ½	M.E.	F.—S. Trig. & Alg... 36 F.—Anal. & Alg.....108 S.—Calc. 90	234	3514	621	0	126	4555	5.2
* [17] Cornell University ..... Ithaca, N. Y.	1868	C.E., M.E., E.E., N.A., A.	Alg. ..... 2 P. & S. Geom. 1½ P. Trig. .... ½	M.E.	F.—Anal. ..... 72 F.—Calc. 126	198	1872	414	0	162	2646	7.5
Manhattan College ..... New York, N. Y.	1888	C.E.	Alg. ..... 1½ P. & S. Geom. 1½ P. Trig. .... ½	C.E.	F.—P. Anal. .....100 S.—Calc. 120	220	2060	720	720	320	4040	5.4

[16] Summer work counted in hours given.
[17] Several specialized courses offered under those mentioned.

The New York University..... New York, N. Y.	1862	M.E., C.E., Ch.	Alg. ..... 2 M.E. P. & S. Geom. 1½ P. & S. Trig. ½	F.—Alg., Rev. Trig.. 72 F.—P. Anal. ..... 90 S.—Calc., S. Anal., Dif. Eqs. 108	270	1584	504	378	90	2826	9.5
Rensselaer Polytechnic Institute Troy, N. Y.	1825	C.E., E.E., M.E.	Alg. ..... 1½ P. & S. Geom. 1½								
[18] Syracuse University ..... Syracuse, N. Y.	1902	C.E., E.E., M.E.	Alg. .....1½ P. & S. Geom. 1½	F.—P. & S. Trig..... 72 F.—P. & S. Anal., H.P.C. .....126 F.—Calc. ..... 36 S.—Calc. 108	342	1998	378	108	0	2826	12.1
Union College ..... Schenectady, N. Y	1845	General, Sani., E.E.	Alg. ..... 1½ Gen. P. & S. Geom. 1½ Trig. ..... ½	F.—Alg. ..... 48 F.—Anal. ..... 84 F.—Calc. ..... 24 F.—Mensuration .... 24 F.—Trig. ..... 12 S.—Calc. ..... 96 J.—S. Trig. & Astra. 36 S.—Least Squares.... 36	360	1260	360	360	216	2556	14.1
* N. Car. Coll. of Agri. and Mechanics Arts ..... West Raleigh, N. C.	1897	E.E., C.E., Mi., A.	Arith. .... E.E. Alg. 1½	F.—Alg. ..... 96 F.—Geom. ..... 84 S.—Geom. ..... 70 S.—Alg. ..... 33 S.—Trig. ..... 77 J.—Anal. .....125 J.—Calc. ..... 55 S.—Calc. 28	568	1440	396	504	144	3052	18.6
University of North Carolina.. Chapel Hill, N. C.	1902	C.E., E.E., Ch., Mi.	P. Geom..... 1 E.E. Alg. 1½	F.—Alg., S. Geom., Trig., Log. ...144 S.—Conics, Calc. ....108 J.—Calc., Mechanics 108	360	1152	360	432	0	2304	15.6

[18] Began C.E. course in 1878 which was abandoned in 1889. This is the only institution to abandon an Engineering Course.

Institutions.	Date	Courses Given.	Entrance Requirements.	Course Scheduled.	Mathematical Course.	M.	P.	Sp.	L.	Mi.	T.	%M.
* North Dakota Agri. College.. Fargo, N. Dak.	1890	M.E., C.E.	Alg. ..... 1½ P. Geom..... 1	M.E.	F.—S. Geom. ..... 60 F.—Alg. .....120 F.—P. Trig. ..... 48 S.—Anal. ..... 60 S.—Calc. 120	408	1068	564	360	492	2892	14.1
[19] University of North Dakota.. Grand Forks, N. Dak.	1890	Mi., M.E., E.E., C.E.	Alg. ..... 1 P. & S. Geom. 1½	M.E.	F.—P. Trig. ..... 48 F.—Alg. combined with Anal..... 96 S.—Calc. 144	288	1548	432	180	0	2448	11.3
Case School of Science ..... Cleveland, O.	1881	C.E., M.E., E.E., Mi., Met., Ch.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Alg. ..... 45 F.—Trig. ..... 70 F.—Anal. ..... 55 S.—P. & S. Anal., H.P.C. ..... 45 S.—Calc. 125	340	1858	476	476	6	3156	10.8
* [20] Ohio State University..... Columbus, O.	1873	C.E., E.E., M.E., Ch., Mi., A., Ceramics.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Alg. ..... 60 F.—Trig. ..... 60 F.—Anal. ..... 60 S.—Calc. .....180 J.—Dif. Eqs. 12	372	1732	360	216	156	2836	13.1
[21] Ohio University ..... Athens, O.	1904	C.E., E.E.	Alg. ..... 1 P. Geom..... 1	E.E.	F.—S. Geom. ..... 48 F.—Alg. ..... 48 F.—P. Trig. ..... 48 S.—Anal. ..... 48 S.—Calc. ..... 48 S.—S. Trig. 48	288	912	120	0	0	1320	21.8 or 10.9

[19] Shop work is advised for summers.
[20] Four weeks shop work during summer counted in hours.
[21] Only two years course offered.

[22] University of Cincinnati..... Cincinnati, O.	1900	C.E., M.E., E.E., Ch.	Alg. ..... 1½ M.E. P. & S. Geom. 1½	F.—Alg., P. Trig..... 72 F.—P. Anal. ..... 72 S.—Calc. 144	288	1904	468	324	24	3008	9.0
* Okla. Agri. and Mech. Coll... Stilwater, Okla.	1898	C.E., E.E., M.E.	Alg. ..... 1 M.E.	F.—Alg. ..... 96 F.—P. & S. Geom....156 S.—P. & S. Geom.... 48 S.—Alg. ..... 36 S.—Trig. ..... 84 J.—Anal. ..... 60 J.—Calc. 120	600	1488	256	180	120	2640	22.7
University of Oklahoma..... Norman, Okla.	1894	C.E., M.E., E.E.	Alg. ..... 1½ M.E. P. & S. Geom. 1½	F.—Col. Alg. ..... 54 F.—Trig. ..... 54 S.—P. & S. Anal..... 72 S.—Calc. 108	288	1494	396	108	234	2520	11.5
* Oregon Agricultural College.. Corvallis, Ore.	1889	C.E., M.E., E.E., Mi.	Alg. ..... 1½ M.E. P. & S. Geom. 1½	F.—Alg. .....126 F.—Trig. ..... 54 S.—P. Anal. ..... 54 S.—Calc. 126	360	1124	288	540	180	2592	13.8
University of Oregon ..... Eugene, Ore.	1895	C.E., M.E., E.E., Mi.	Alg. ..... 1½ M.E. P. & S. Geom. 1½	F.—Alg., Trig., Anal.180 S.—Calc. 180	360	1296	360	36	108	2160	16.6
Alleghany College ..... Meadville, Pa.	1886	C.E.	Alg. ..... 1½ C.E. P. & S. Geom. 1½	F.—Alg. ..... 72 F.—P. & S. Trig..... 72 S.—P. & S. Anal., Calc. 144	288	792	468	342	270	2160	13.3
Bucknell College ..... Lewisburg, Pa.	1903	C.E., E.E.	Alg. ..... 1½ E.E. P. & S. Geom. 1½	F.—P. Trig. ..... 60 F.—Alg. ..... 60 F.—Anal. ..... 60 S.—Calc. 120	300	1296	360	300	84	2340	12.9
Grove City College ..... Grove City, Pa.	1901	C.E., M.E.	Alg. ..... 1½ M.E. P. & S. Geom. 1½	F.—P. Trig. ..... 60 F.—Alg. ..... 60 S.—Anal. ..... 60 S.—Calc. 120	300	592	576	756	528	2752	10.9

[22] Four weeks for three summers counted in hours.

Institutions.	Date	Courses Given.	Entrance Requirements.	Course Scheduled.	Mathematical Course.	M.	P.	Sp.	L.	Mi.	T.	%M.
Haverford College ..... Haverford, Pa.	1880	M.E.	Alg. ..... 1½ P. & S. Geom. 1½ P. Trig. .... ½	M.E.	F.—Alg. ..... 96 F.—Spec. S. Geom., Trig. ..... 18 S.—P. Anal. .....108 S.—Calc. ..... 72 J.—Calc. ..... 54 J.—S. Anal., Dif. Eqs 54	402	738	325	210	480	2155	18.6
[23] Lafayette College ..... Easton, Pa.	1866	C.E., E.E., Mi.	Alg. ..... 1½ P. & S. Geom. 1½ P. Trig. .... ½	E.E.	F.—Alg. ..... 48 F.—P. Trig. ..... 60 F.—S. Trig., Anal.... 60 S.—Anal. ..... 72 S.—Calc. ..... 48 S.—Dif. Eqs., Least Squares 36	324	1320	408	288	168	3516	9.2
[24] Lehigh University ..... S. Bethlehem, Pa.	1866	C.E., M.E., Mi., Met., E.E., Ch., Electro-Met.	Alg. ..... 2 P. & S. Geom. 1½ Trig. ..... ½	M.E.	F.—P. & S. Anal....108 F.—Calc. 108	216	1530	378	450	126	2700	8.0
* Pennsylvania State College... State College, Pa.	1863	C.E., E.E., Mi.	Alg. ..... 2 P. & S. Geom. 1½	M.E.	F.—Trig. ..... 82 F.—Anal. ..... 82 S.—Alg. ..... 33 S.—Calc. 132	329	1504	288	324	270	2715	12.1
Swarthmore College ..... Swarthmore, Pa.	1872	C.E., M.E., E.E.	Alg. ..... 1½ P. Geom..... 1	M.E.	F.—S. Geom. ..... 54 F.—Alg. ..... 72 F.—Trig. ..... 54 S.—P. Anal. ..... 54 S.—Calc. ..... 54 J.—Calc. ..... 54 J.—Dif. Eqs. 54	396	1332	504	432	108	2772	14.3

[23] Six weeks summer work counted in hours.
[24] Three summer tours of inspection of four weeks each not counted in hours.

Towne Scientific School..... University of Pennsylvania. Philadelphia, Pa.	1875	M.E., E.E., C.E., A., Ch.	Alg. ..... 1½ P. & S. Geom. 1½ P. Trig. .... ½	M.E.	F.—P. & S. Trig..... 72 F.—Alg. ..... 72 S.—Anal. ..... 90 S.—Calc. 90	324	1674	432	396	0	2826	11.5
[5] Carnegie Technical School... Pittsburg, Pa.	1905	C.E., Ch., E.E., M.E., Met., Mi.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	Alg. ..... 54 Trig. ..... 63 Conics, H.P.C. ..... 54 Calc. 117	288	1722	294	288	252	2844	10.2
Washington & Jefferson Coll... Washington, Pa.	1898	General.	Alg. ..... 1½ P. & S. Geom. 1½	Gen.	F.—Alg. Dec. Geom., P. Trig. .....144 S.—P. Anal., S. Trig., Mechanics ....108 J.—Calc. ..... 72 Sr.—Calc. 72	396	468	486	468	386	2204	18.0
[20] University of Pittsburg..... Pittsburg, Pa.	1865	C.E., M.E., E.E., Ch.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Alg. ..... 54 F.—P. & S. Trig. .... 54 F.—P. Anal. ..... 72 S.—Calc. 108	288	1314	306	396	306	2610	11.0
Brown University ..... Providence, R. I.	1849	C.E., E.E., M.E.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Alg. ..... 72 F.—P. & S. Trig..... 72 F.—Anal. ..... 72 S.—Calc. 132	348	1548	216	216	216	2544	13.7
* Rhode Island State College... Kingston, R. I.	1892	M.E., E.E., C.E., Ch.	Alg. ..... 1½ P. & S. Geom. 1	M.E.	F.—Alg., S. Geom.... 90 F.—P. Trig., Anal.... 90 S.—Anal., Calc. ..... 90 S.—Calc. 90	360	1539	414	414	414	3141	11.4
* Clemson Agricultural College. Clemson College, S. C.	1893	M.E., E.E., C.E., Textile.	Arith. & Alg. 1	M.E.	F.—Alg., Geom.....180 S.—P.& S. Trig., Alg., P.& S. Anal., H.P.C. .....132 J.—Calc. 180	492	1092	364	408	524	2880	17.1

[20] Mathematics courses are not scheduled for any particular year in Catalog.
[26] Changed name from Western University of Pennsylvania in 1908.

Institutions.	Date	Courses Given.	Entrance Requirements.	Course Scheduled.	Mathematical Course.	M.	P.	Sp.	L.	Mi.	T.	%M.
Newberry College ..... Newberry, S. C.	1904	M.E., E.E.	Alg.	1½ M.E.	F.—Alg., Trig.....144 F.—Geom. .....144 S.—Trig., Surv., Anal 108 J.—Equas., Calc.....144 Sr.—Calc. 144	684	1308	516	504	144	3156	21.7
University of South Carolina.. Columbia, S. C.	1880	C.E., E.E.	Alg. ..... 1½ E.E. P. Geom..... ½		F.—Alg., Geom.....180 S.—P.& S. Trig.,Anal 108 J.—S. Anal., Ele. Calc. .....108 Sr.—Calc. 108	504	432	594	432	882	2844	17.7
* S. Dak. State Coll. of Agri. & Mechanics Arts ..... Brookings, S. Dak.	1885	M.E., E.E., C.E.	Alg. ..... 1½ M.E. P. Geom..... 1		F.—S. Geom. ..... 54 F.—P. & S. Trig..... 72 F.—Alg. ..... 54 S.—Anal., Calc. 180	360	1474	432	360	126	2772	12.9
" S. Dak. School of Mines..... Rapid City, S. Dak.	1885	Mi., Met.	Alg. ..... 1½Mi. P. & S. Geom. 1½		F.—P. & S. Trig..... 90 F.—P. Anal., H.P.C.. 90 S.—Calc. with topics of Alg. & S. Anal. 180	360	1980	900	144	0	3384	10.6
University of South Dakota.... Vermillion, S. Dak.	1901	M.E., C.E., E.E., Ch.	Alg. ..... 1½ M.E. P. & S. Geom. 1½		F.—Trig. ..... 54 F.—Alg. ..... 54 F.—Anal. ..... 90 S.—Calc. 108	306	1628	432	54	0	2520	12.1
Cumberland University ..... Lebanon, Tenn.	1852	C.E., A., Top.	Alg. ..... 1½ C.E. P. & S. Geom. 1½			...	....	...	...	...	....	....

" Two or three weeks of summer inspection tours not counted in hours.

University of the South..... Sewanee, Tenn.	1870	C.E.	Alg. ..... 1½ C.E. P. & S. Geom. 1½	F.—P. & S. Trig. .... 54 F.—Alg. ..... 54 S.—P. Anal. ..... 54 S.—Anal., Eqs. ..... 54 J.—Calc. 108	324	504	540	324	432	2124	15.2
*University of Tennessee..... Knoxville, Tenn.	1869	C.E., E.E., M.E., Mi., Ch.	Alg. ..... 1½ M.E. P. Geom..... 1	F.—Alg. ..... 60 F.—Geom. ..... 60 F.—Trig. ..... 60 S.—Anal. ..... 36 S.—Calc. 72	288	1692	468	288	108	2844	10.1
Vanderbilt University ..... Nashville, Tenn.	1876	C.E., M.E., E.E., Ch., Mi.	Alg. ..... 1½ M.E. P. & S. Geom. 1½	F.—Trig., P. Anal., Calc. .....144 S.—Calc., S. Anal., Alg. 108	252	1876	468	288	216	3100	8.1
*Agri. & Mech. Coll. of Texas. College Station, Tex.	1876	M.E., C.E., E.E., A.E., Textile.	Alg. .....1 M.E.	F.—P. & S. Geom....144 F.—Alg. ..... 72 S.—P. & S. Trig..... 72 S.—Alg. ..... 36 S.—P. & S. Anal.....108 J.—Calc. .....108 Sr.—Applications Calc. to Eng... 72	612	1228	502	216	300	2858	21.8
University of Texas..... Austin, Tex.	1881	C.E., E.E., Mi., A.E., Sani.	Alg. ..... 1½ M.E. P. Geom..... 1	F.—Trig. ..... 36 F.—Alg. ..... 36 F.—Geom. ..... 36 F.—Anal. ..... 72 S.—Calc. 108	288	1260	432	180	108	2268	10.8
University of Utah..... Salt Lake City, Utah.	1895	M.E., E.E., C.E., Ch., Mi., Gen.	Alg. ..... 1½ M.E. P. & S. Geom. 1½ P. Trig. .... ½	F.—Alg. ..... 72 F.—Anal. ..... 72 S.—Calc. 144	288	1602	468	72	0	2430	11.8

Institutions.	Date	Courses Given.	Entrance Requirements.	Course Scheduled.	Mathematical Course.	M.	P.	Sp.	L.	Mi.	T.	%M.
Norwich College ..... Northfield, Vt.	1891	C.E., E.E., Ch.	Alg. ..... 1½ P. & S. Geom. 1½	E.E.	F.—Alg. ..... 65 F.—P. & S. Trig..... 60 F.—P. & S. Anal..... 55 S.—P. & S. Anal..... 26 S.—Calc. 120	326	1506	348	324	441	2945	11.1
* Univ. of Vt., and Agri. Coll... Burlington, Vt.	1865	C.E., M.E., E.E.	Alg. ..... 1½ P. & S. Geom. 1½	M.E.	F.—Alg. ..... 72 F.—Trig. ..... 36 F.—Anal. ..... 72 S.—Calc. 108	288	1728	360	396	108	2880	10.0
University of Virginia..... Charlottesville, Va.	1867	C.E., E.E., M.E., Mi., Ch.	Alg. ..... 2 P. & S. Geom. 1½ P. Trig. .... ½	M.E.	F.—Trig., Alg., Ele. Anal. .....198 S.—Anal. & Calc.....180	368	1643	569	0	0	2580	14.3
[28] Virginia Military Institute.... Lexington, Va.	1839	C.E., E.E., Ch.	Alg. 1	E.E.	F.—Alg. .....108 F.—P. & S. Geom.... 99 F.—P. Trig..... 9 S.—Logs, Trig. .....108 S.—Anal. .....108 J.—Calc. 216	648	773	378	648	378	2825	23.0
* Virginia Polytechnic Institute. Blacksburg, Va.	1893	C.E., E.E., M.E., Mi., Met.	Alg. ..... 1 P. Geom. .... ½	M.E.	F.—Alg. ..... 72 F.—Geom. ..... 72 F.—Trig. ..... 72 S.—Anal. .....108 J.—Calc. 108	432	1472	652	756	288	3600	12.0
Washington & Lee University.. Lexington, Va.	1870	C.E., Mi.	Alg. ..... 1 P. Geom..... 1	C.E.	F.—S. Geom., Alg., P. Trig. .....144 S.—S. Trig., Anal., Ele. Calc. .....180 J.—Calc., Dif. Eqs...180	504	756	720	324	108	2412	20.9

[28] School in session 41 weeks but computations based on 36 weeks only.

* State College of Washington. Pullman, Wash.	1892	M.E., E.E., Mi.	Alg. ..... 1½ M.E. P. & S. Geom. 1½	F.—Adv. Geom. ..... 36 F.—Trig. ..... 90 F.—Anal. ..... 54 F.—Alg. ..... 90 S.—Calc. ..... 90 J.—Calc. 90	450	1314	180	450	486	2880	15.6
University of Washington..... Seattle, Wash.	1895	C.E., E.E., M.E., Mi., Ch.	Alg. ..... 1½ M.E. P. & S. Geom. 1½	F.—P. Trig., Alg. ... 72 F.—P. Anal., Alg. ... 72 S.—S. Anal. ..... 36 S.—Calc. 144	324	1482	378	72	192	2448	13.2
* West Virginia University.... Morgantown, W. Va.	1888	C.E., E.E., M.E., Mi.	Alg. ..... 1½ M.E. P. & S. Geom. 1½	F.—Alg. ..... 60 F.—P. Trig. ..... 60 F.—P. Anal. ..... 60 S.—Calc. 120	300	1764	396	180	0	2640	11.4
* [29] University of Wisconsin.... Madison, Wis.	1870	C.E., M.E., Ch., E.E., Sani., Mi., General, Electro-Ch.	Alg. ..... 1½ M.E. P. & S. Geom..1½	F.—Alg., Trig., Anal 180 S.—Calc. 162	342	1962	288	252	0	2848	12.0
* University of Wyoming..... Laramie, Wyo.	1890	M.E., C.E., Mi., Irrig.	Alg. ..... 1½ M.E. P. & S. Geom..1½	F.—Alg., P. & S. Trig 90 F.—P. & S. Anal..... 90 S.—Calc. 180	360	1494	360	108	162	2484	14.5
[30] U. S. Military Academy..... West Point, N. Y.	1802	Military.	Alg. ..... 1½ Mil. P. Geom..... 1	Preliminary—Alg. ... 52 Preliminary—Geom. ..49 F.—Alg. ..... 60 F.—Trig. ..... 41 F.—Conics ..... 75 F.—S. Anal ..... 29 S.—Calc. 127	433	962	514	513	100	2622	16.5
U. S. Naval Academy..... Annapolis Md.	1845	Naval.	Arith. ....Naval Alg. ..... 1½ P. Geom..... 1	F.—Alg., Logs., Geom. .....108 F.—Trig. .....108 S.—Anal., Calc. ..... 90 S.—Calc., S. Trig., Projectiles.....108	414	1406	120	476	16	2432	17.0

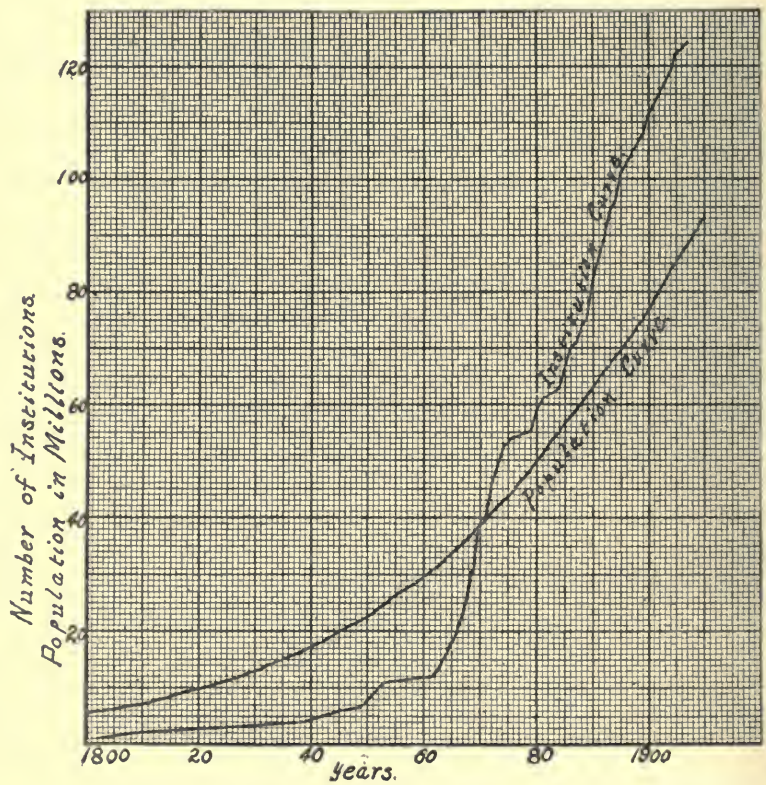
[29] Summer work equivalent to 4 semester hours counted in hours.
[30] Professional includes 468 hours military engineering.

V. COLLECTED RESULTS OF THE TABLES.

The facts given in detail in the tables are collected below in summary form for certain important phases of the work.

1) Growth of Engineering Activity.

The increase in the number of institutions offering courses in engineering from the founding of the United States Military Acad-



emy in 1802 until 1908 is shown in the accompanying graph. Dates are plotted as abscissae and the total number of institutions giving courses in engineering as ordinates. The population curve for the United States covering this period is also plotted to the same axes.

2) ENTRANCE REQUIREMENTS. (*i*) *Time for Various Subjects.*

The following table gives the numerical data of the entrance requirements of the one hundred and thirty institutions recorded on pages 21 to 41 as presented in collected form:

	No. of Institutions.
Arithmetic	11
Algebra.	
None	1
One-half year	1
One year	14
One and one-half years	95
Two years	14
Three years	1
Plane Geometry.	
None	9
One-half year	3
One year	114
Solid Geometry.	
None	39
One-half year	87
Trigonometry.	
One-half year	24
All work in mathematics (thru the calculus)	1

Whether plane trigonometry is required for admission or accepted as one of the entrance electives, the time asked for it is in all cases one-half year. Nine of the institutions requiring trigonometry for entrance give a review of it during the college course. The time required for the completion of plane geometry is uniformly one year and for the completion of solid geometry one-half year. The requirements in algebra vary greatly both as to time and as to work, a fact which is greatly to be deplored as it is the lack of preparation in algebra that gives the student of engineering his greatest difficulties, as will be fully discussed a little later. It will be seen that the time requirements vary from nothing at all to three full years; the great majority or 73%, however, require one and one-half years. Of the fourteen which require two years seven include college algebra and the remaining seven a half year's review during the last year of the preparatory course.

(*ii*) *Requirements of a Majority of the Institutions.*

The preceding table shows that the great majority of the institutions require for entrance one and one-half years of algebra, one year of plane geometry and one-half year of solid geometry.

3) CURRICULUM—(*i*) *Position of Subject.*

The following table shows the number of institutions in which the various subjects are given at the stage in the curriculum that is named:

	No. of Institutions.
Trigonometry.	
For entrance	13
Completed first year	94
Completed second year	11
Completed third year	1
Algebra.	
All for entrance	13
Completed first year	95
Completed second year	12
Completed third year	2
Analytic Geometry.	
All for entrance	1
Completed first year	59
Completed second year	59
Completed after second year	4
Calculus.	
All for entrance	1
Completed first year	2
Begun first year, completed second year	14
All in second year	75
Completed after second year	31

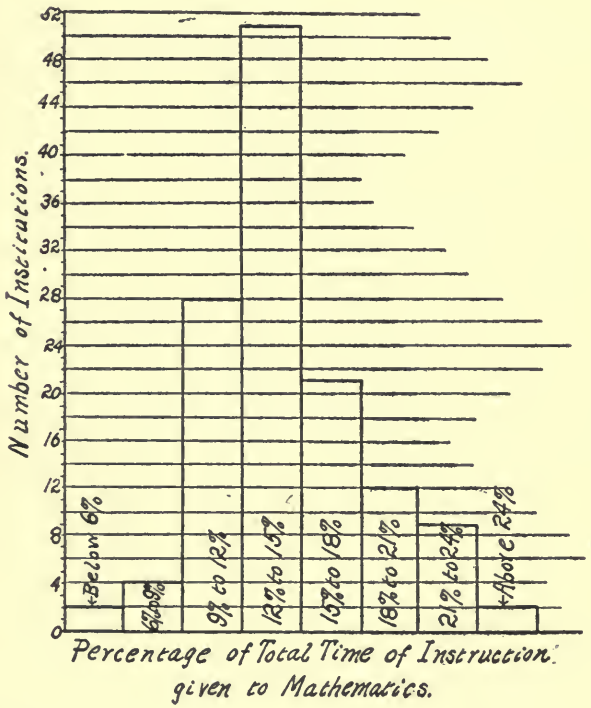
Differential equations are mentioned as separate courses by 20 of the institutions, and theory of equations by 7. As a rule the latter is included in the work scheduled under algebra and does not receive special mention. The theory of least squares is included by 4. Only 39 mention offering spherical trigonometry, the others either specify plane trigonometry only or give no hint as to the

contents of the course. In the same way only 40 specify giving solid analytic geometry.

The discrepancies in the above table are due to two things: first, that no data could be obtained from 7 of the institutions, and secondly, that 11 of the institutions give special courses[3] in mathematics from which the subjects mentioned could not very readily be differentiated.

(*ii*) *General Trend of Position of Subjects in Curriculum.*

From the above table and that regarding entrance requirements on page 43, we see that there is a general trend towards giving trigonometry, algebra and analytic geometry in the first year of the college course, and all of the calculus in the second year.



[3] Chap. III.

(iii) Comparison of Time for Various Subjects.

The accompanying cuts, which are self explanatory, give a comparison of the time devoted to the various subjects, trigonometry, algebra, analytic geometry and the calculus by the various institutions recorded on pages 21 to 41.

4) Percentage of Time Given to Mathematics.

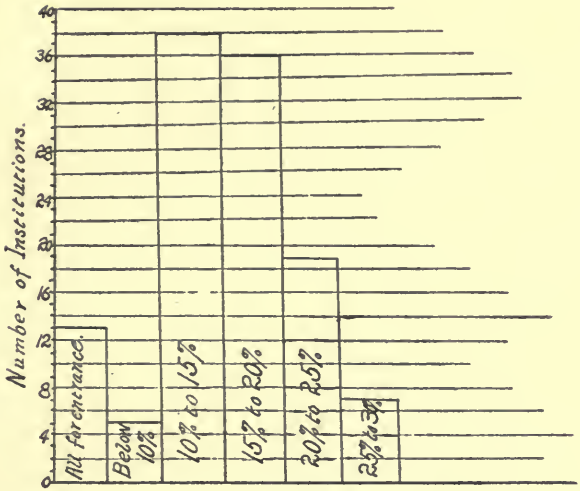
The percentages of the total time of instruction in the four collegiate years given to mathematics are exhibited in the graph on page 45.

5) Discrepancies and Their Causes.

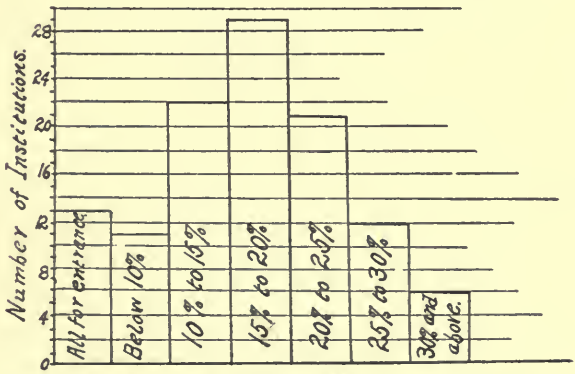
The great variety in the number of hours recorded in the columns shows the lack of uniformity among the engineering colleges of the United States. Some of the more noticeable ones will here be briefly considered. Where entrance requirements in mathematics are low the time devoted to the collegiate work is correspondingly large. The wide range in the amount of time given miscellaneous subjects is due to the fact that a varying degree of importance is attached to culture as compared with professional subjects, or more often that the engineering courses are in varying degrees modifications of the general science courses. The great variation in the number of hours recorded under "P," "Sp" and especially under "T," is accounted for in two ways. The first and real factor is, that different relative values are given to recitations, lectures, laboratory and industrial periods. By "industrial periods" is meant such work as drawing, shop-work, etc., which requires no outside preparation. The following four ratios of equivalence of the different kinds of work are found in use:

No. of hours in	Mathematics	Laboratory	Industrial
i	1	2	3
ii	1	2	2½
iii	1	2½	2½
iv	1	2	2

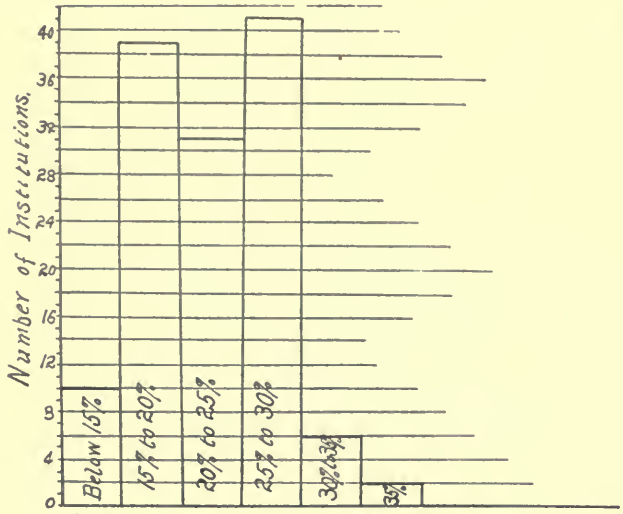
In the majority of cases the institution which used the first ratio recorded the lowest number of hours under "professional"



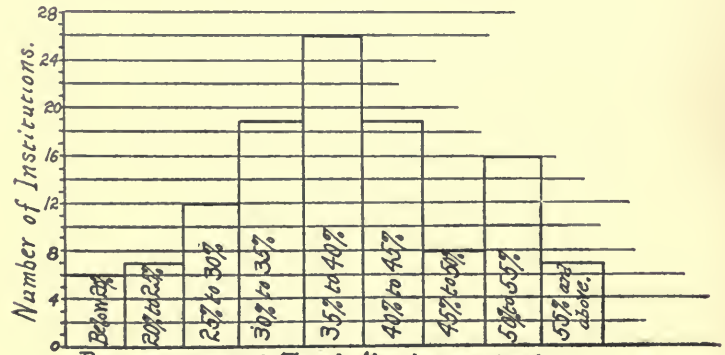
Percentage of Total Mathematical Instruction given to Trigonometry.



Percentage of Total Mathematical Instruction given to College Algebra.



Percentage of Total Mathematical Instruction given to Analytic Geometry.



Percentage of Total Mathematical Instruction given to the Calculus.

subjects, which increased for the second, third and fourth ratios in that order of sequence. The second cause for variation is merely a clerical one, arising from insufficient data in the various catalogs.

VI. PERCENTAGE OF TIME GIVEN VARIOUS GROUPS OF WORK.

One further computation was made: the percentages were computed for each of the divisions "mathematics," "professional," "semi-professional," "language" and "miscellaneous" by adding the total time recorded for each. The results were as follows:

Professional 52.3%	Semi-professional 15.0%
Mathematics 13.7%	Language 11.0%
Miscellaneous 8.0%	

VII. SUMMARY.

There are one hundred and thirty instituions giving degrees for work in engineering, one hundred and twenty-one of which have been organized since 1860; the general entrance requirements are one and one-half years algebra, one year plane geometry and one-half year of solid geometry with trigonometry required by 18% of the institutions; the general trend of collegiate mathematics is to complete trigonometry, algebra and analytic geometry the first year and to devote the second year to calculus, except that a few institutions are beginning to introduce the calculus into the first year; there is great variation in the allotment of time to the various departments and especially to the department of mathematics where the allotment varies from 5% to 23% of the total time; mathematics receives only one-fourth as much time as the professional studies and only slightly more than those in language.

CHAPTER III.

RECENT MODIFICATIONS IN THE WORK IN MATHEMATICS FOR STUDENTS OF ENGINEERING.

I. CONTENTS OF THE CHAPTER.

As already noted[1] some of the engineering colleges of the United States have discarded the formal division of the work in mathematics into subjects separated by sharply defined lines, and some institutions have added new features, such as laboratory work in curve tracing, special courses in computation, etc. A treatment of this modified work will be the object of the present chapter.

II. GENERAL NATURE OF THE MODIFIED WORK IN MATHEMATICS.

This departure from the old lines aims to arrange the work in mathematics for the student of engineering to fit his special needs, and is, as will be seen, in full conformity with the so-called "Perry Movement."[2] In 1902 Prof. Moore advocated an immediate reform in the teaching of elementary collegiate mathematics so as to bring it

[1] Chap. II, pp. 21 to 41.

[2] It may be best to give Prof. Perry's idea of the necessary reform in the teaching of mathematics in his own words: "Newton employed geometrical conics in his astronomical studies and so mechanics was developed, and therefore it is that every young engineer must study mechanics thru astronomy, and he dares not think of differential calculus till he has finished geometrical conics. The young applier of physics, the engineer, needs a teaching of mathematics which will make his mathematical knowledge part of his mental machinery, which he will use as readily and as certainly as a bird uses its wings—." John Perry, On the Teaching of Mathematics, Nature, vol. 26. p, 319.

within the realm of reality.[3] Such modifications have been worked out in a number of engineering institutions where the object has been to build up a logical course in mathematics based upon the future needs of the student.[4] These modifications are as yet in the evolutionary stage but have as their object the treatment of matheematics as a scientific "weapon."[5]

III. SOURCES OF DATA.

Twenty-two of the institutions considered in Chapter II are found to have established, to a greater or less extent, such modified work in mathematics. In order to secure more complete information regarding this work than is found in the catalogs a questionaire letter was sent to each institution, uniformly containing the following questions together with such others as were suggested by the separate catalogs.

[3] "Just as the secondary schools should begin to reform without waiting for the improvement of the primary schools, so the elementary collegiate courses should be modified at once without waiting for the reform of the secondary schools." "— a feeling that mathematics is indeed itself a fundamental reality of the domain of thought and not merely a matter of symbols and arbitrary rules and conventions." Presidential Address at the Annual Meeting of the American Mathematical Society. Bull. Am. Math. Soc., 1903, pp. 1-24.

[4] "Some technical schools have made radical changes in the mathematical work of the college course. Those which have worked out the matter independently have arrived at very similar results. The changes consist for the most part of modifications that naturally follow from the movement in the secondary schools. The tendency is to blot out entirely the present lines of distinction between algebra, trigonometry and analytic geometry. These changes have involved, not only the early and constant use of graphical methods and abundant use of numerical data, but have also included the early consideration and application of vector analysis. Our most treasonable act is the deposing of the conic sections, which have been the reigning family for so many years, from their exalted place in our course of study. This gives an opportunity for the more abundant enrichment of the course and a better comprehension of those things which the student of applied science needs. Such a course not only conforms more nearly to the actual needs of the students, but it also has the advantage of being more logical and scientific than our old course. As a matter of fact all that the engineering student learns in his usual course in mathematics is a simple comprehension of the properties of the algebraic functions of a real variable. Indeed we might define the work of the engineering student, during his first two years of mathematics, as the study of the exponential function of the real variable." Chas S. Slichter, The Improvement of the Freshman Year of Mathematical Instruction in Technical Schools, Proceedings of the Society for the Promotion of Engineering Education, vol. 14, p. 146.

[5] "—a most powerful weapon with which to unlock the mysteries of Nature." John Perry, Address before the Section of Educational Science of the British Association, at Glasgow, 1902.

1) Questionaire Letter.

(*i*) In what way does the work differ from that usually given in a general science or arts course?

(*ii*) What is the material used and from what source is it obtained?

(*iii*) What is the particular object sought?

(*iv*) When was this instruction with special reference to engineering begun, and what suggested the same?

While some replies were quite brief, others were very comprehensive and described fully the work given. No replies were received from seven of the institutions written to, in which cases reference will be made to the statements found in the catalogs wherever usable.

IV. SYNOPSIS OF PRINCIPAL FEATURES OF MODIFIED WORK.

The following synopsis shows for twenty of the above institutions separately the principal features of the instruction ascertained as just specified. The statements are arranged according to similarity of their principal features, and are followed by a short summary.

Institution No. 1. Coherent Course of Essential Elements.

The reply to the letter of inquiry suggested the consultation of "A Course in Mathematics," by Woods and Bailey, the first volume of which is used as a text during the freshman year.

It may be well to quote from the preface in regard to the aims and objects of the authors: "This book is the first volume of a course in mathematics designed to present in a consecutive manner an amount of material generally given in distinct courses under the various names algebra, analytical geometry, differential and integral calculus, and differential equations.....In arranging the material, however, the traditional division of mathematics into distinct subjects is disregarded, and the principles of each subject are introduced as needed and the subjects developed together. The objects are to give the student a better grasp of mathematics as a whole, and of interdependence of its various parts, and to accustom him to use, in

later applications, the methods best adapted to the problem in hand. At the same time a decided advantage is gained in the introduction of the principles of analytic geometry and the calculus earlier than is usual. In this way these subjects are studied longer than is otherwise possible, thus leading to greater familiarity with their methods and greater freedom and skill in their application."

(*i*) *Study and Use of Curves.*

Accordingly the work in mathematics in this institution appears to be as follows: A few preliminary lessons are devoted to determinants, considering only evaluation of determinants, their use in the solution of linear simultaneous equations, eliminants and the testing of equations for common roots.

Some work on the different kinds of numbers and functions, introduces a graphic study of polynomials. After a short consideration of derivatives of polynomials there follows a discussion of curves, considerable time being devoted to conics but only as a species of curves in general, where such questions as tangents, normals, diameters, cusps, asymptotes, branches, intersections of curves and systems of curves are taken up. The general equation of the second degree is not taken up at all. Polar and parametric representation of curves are taken up as separate topics and applied to problems for which they are most servicable. Solid analytic geometry is deferred until the second year.

(*ii*) *Early Introduction of the Calculus.*

Derivatives of polynomials are introduced quite early in the course thru the idea of the slope, and are immediately applied to problems on tangents, and maxima and minima. A more complete treatment follows the study of curves as mentioned above, where simple problems of physics are also considered. Quite a number of transcendental functions are studied towards the close of the course, which ends with a brief treatment of curvature, evolutes and involutes of curves.

INSTITUTION NO. 2. MAXIMUM USABLE MATHEMATICS IN FIRST YEAR.

In this institution a course called "An Introduction to Modern Mathematics" is given the first year the aim of which is a "desire to start the student with as much mathematics as possible in the

freshman year so that he can grasp work required in the professional schools as early as possible." It includes "much that was formerly taught under the head of Theory of Equations, combined with an early introduction to the elements of analytic geometry and calculus, —while not anticipating the calculus to any extent." "A Course in Mathematics" is also used as a text here and supplemented by Wentworth's Analytic Geometry", especially for problems in solid analytic geometry.

(*i*) *Origin of Course.*

This course was introduced in 1907 as many of the students came well prepared in trigonometry and higher algebra so that such a course could be taken advantageously, both as to time and preparation. A rigorous examination in trigonometry must be passed by all students admitted to it. The class is divided into sections based upon the ability of the students.

INSTITUTION NO. 3. MATHEMATICAL SCHEDULE A COHERENT WHOLE.

The catalog states, "The aim of the instruction in mathematics is to present the subject so that the student may obtain a thoro working knowledge of these principles which he needs to know when he becomes an engineer. It is recognized that such knowledge can best be obtained by an exercise of the observational faculties of the student, by treating the subject as one coherent whole rather than a series of more or less disconnected subjects, and by frequent application of the principles taught to problems in engineering. For this reason laboratory methods are sometimes employed—and the subjects taught are so arranged that each is a help in the development of the others. We give the calculus in the freshman year in order that the men may early become acquainted with the calculus method so as to use the subject more intelligently in the many cases which arise during the sophomore year, in physics, kinematics, etc."

(*i*) *College Algebra.*

The freshmen take five hours of algebra during the first half of the year, comprising "a review of equations; plotting of curves from equations; a comprehensive treatment of surds, imaginaries, ratio, proportion, variation; the progressions; permutations; com-

binations; determinants; binomial theorem for positive integral powers of the binomial; logarithms; partial fractions; methods of approximations to the roots of equations; building up of equations from the properties of the curves. A special feature of the course is the introduction of a large number of problems and curves similar to those met by the engineers in actual practice in order to drill the student on many algebraic operations peculiar to engineering, especially those for logarithmic computations which are not found in the ordinary algebra. By stating the actual engineering problem, with the physical law, we try and do, to a great extent, arouse the student's interest in the subject of college algebra, a study that is too often found dry and uninteresting."

(*ii*) *Analytic Geometry and Calculus.*

Five hours per week are given during the second semester to analytic geometry and the elements of the calculus, when are considered: "Transformation of coordinates; a systematic treatment of the circle, parabola, ellipse and hyperbola; limits; the ordinary rules for differentiation with application to curve plotting, rates, maxima and minima; the fundamental forms of integration with easy applications to problems in plane areas; those subjects in college algebra and analytical geometry which lend themselves best to the calculus treatment." In analytical geometry the straight line and loci problems receive the principal consideration; for conics only questions of vertices, foci and directrices are taken up. Problems on tangents are treated by the calculus and are not restricted to conics. The various forms of differentiation are taken up together with their applications.

(*iii*) *Material Used.*

The texts on algebra and trigonometry are supplemented by numerous mimeographed problems as mentioned above, of which the following are types:

1) SIMPLIFY $\frac{(3^{2n})^{n-1}}{3^3 \cdot 3^{n-2}} \cdot \frac{(81)^{2-n} \cdot 243^{n(n-1)}}{[(27)^{n-1}]^{n+1}}$

2) EVALUATE $(3.16)^{.142}$ **and** $\sqrt[3]{\frac{\pi^{\frac{1}{2}} \times 3.15 \times .32}{.004 \times .00032}}$

3) SOLVE THE EQUATION $\sqrt{5^x} + \sqrt{5^{-x}} = \frac{29}{10}$

Given that (an expression in X) $\Big/_{b}^{a}$ means the value of the expression when X = a minus its value when X = b, and given e = 2.71828, evaluate

4) LOG $(\sec X + \tan X)\Big/_{0}^{\frac{\pi}{4}}$

5) SOLVE FOR X AND Y $\begin{cases} 4^{3y-2x} = 7^{2x} \\ 3.4^{y+x} = 16^{x} \end{cases}$

6) FIND LOG $\dfrac{\sqrt[4]{5}\ \sqrt[10]{2}}{\sqrt[3]{18\sqrt{2}}}$

7) EXPAND THE DETERMINANT $\begin{vmatrix} 1 & 1 & 1 \\ 3 & 5 & 7 \\ 9 & 8 & 4 \end{vmatrix}$

8) SOLVE $\dfrac{X-8}{X-10} - \dfrac{X-4}{X-6} = \dfrac{X-5}{X-7} - \dfrac{X-7}{X-9}$

Use logarithms in the following problems involving computations.

9) Within the elastic limit, the expression of a spiral due to a weight suspended to one end is proportional to the weight. A spring originally 18.23″ long measures 20.35″ when a weight of 3.2 oz. is attached; what will be its length when the weight is 4.5 oz.?

10) Within the elastic limit, the extension of a rod under tension varies directly as the product of the length and the tension, and inversely as the area of the cross-section. If a round wrought iron rod, 3′ long, .625″ in diameter, is stretched .0057″ by a tension of 6000 lbs., how much will a bar of wrought iron 10′ long and of rectangular cross-section .25″ by .375″ be stretched by a tension of 560 lbs.?

Several problems are given under each of the conditions as stated above.

INSTITUTION NO. 4. A MODIFIED COHERENT SCHEDULE OF SUBJECTS.

Beginning with the work in algebra and continuing thru the calculus the work of this institution is arranged as a systematic whole, while not wholly disregarding the division into the subjects algebra, analytic geometry, differential and integral calculus.

INSTITUTION No. 5. ARRANGEMENT OF MATHEMATICS COURSES FOR FUTURE WORK.

The freshman work begins with a review of surds, imaginaries and quadratic equations, followed by those principles needed in analytic geometry. The whole mathematical schedule is arranged with view to the work taken by the students in their third and fourth years covering such subjects as mechanics, hydraulics, machine design, bridges, etc. This arrangement was reported as meeting with the approval of the professional departments as well as with that of the instructors of mathematics, the only difficulty being that of finding suitable text-books. Trigonometry is given as a formal course in the first year and all that is given of the calculus constitutes the second year's work.

INSTITUTION No. 6. COMBINED COURSE IN ALGEBRA AND TRIGONOMETRY.

A course in algebra and trigonometry is given the first half of the freshman year in which, "the work in algebra deals with topics supplementing the work in trigonometry;" later in the course trigonometry in turn is used in the solution of equations.

(*i*) *Mathematics as a Tool.*

As to the objects and the handling of the course in mathematics an instructor reports:—"We lay stress upon the theory but insist upon familiarity with processes and manipulations of results. For instance, in the study of Taylor's Theorem, we do not go into the remainder form of the theorem, but lay stress upon the actual use of the theorem in the expansion of functions. In the integral calculus we lay less stress upon the derivation of the reduction formulas than upon the use of the tables of integration. We develop the calculus as a tool rather than as a science for its own sake."

INSTITUTION No. 7. ALGEBRA THRUOUT THE COURSE IN MATHEMATICS.

No formal course is given in algebra but the subject is distributed thruout the whole course in mathematics. When a topic is reached that calls for any principles in algebra which are new to the students or which present difficulties, such principles are taken up and elucidated.

INSTITUTION NO. 8. EARLY CALCULUS.

A course in analysis is given the second half of the first year in which are treated some of the advanced topics of algebra and an introduction to the calculus. This is followed by a formal course in analytic geometry, where the principles of the former courses are applied.

INSTITUTION NO. 9. MATHEMATICS AS BASIS OF TECHNICAL TRAINING.

The mathematical courses are conducted by means of well chosen text-books, very little supplementary material being used. Special attention is given to practical problems which are interesting to the students. "The ability to understand and apply mathematical processes readily is the aim, and to this end special emphasis is laid upon two things: elucidation of the principles and drill upon their application, as furnishing the only sure basis for a thorough technical and professional training."

INSTITUTION NO. 10. APPLIED CONTRASTED WITH PURE MATHEMATICS.

The reply from this institution was especially clear in stating the object of their special work in mathematics and its variation from that held in view for the art students. To quote: "The object we have in view is to give the students facility in using mathematics intelligently in solving engineering problems. In the A.B. course, on the other hand, we aim to give the student a broad view of the field of mathematics as a branch of human knowledge and some of its big results. In brief, these aims animate our methods."

For example the engineer "has to study the elements of accuracy in testing; this demands partial derivatives, etc." For the A.B. student, "Partial derivatives come under the general head of space differentiation and involve a grip of the real meaning of calculus in its widest sense."

"I know that there are some that contend that the engineer should have this broad insight into mathematical ideas, but the thing is not feasible. He is primarily interested in material construction, and he cannot focus his attention on two things. He is, and ought to be, concerned with effective construction. In handling this,

mathematics is to him a different thing from what it is to one who is interested in the harmonies of numbers and space. I do not believe there is an engineering mathematics, but I do believe there is an engineer's use of mathematics.

"Therefore our object primarily is to teach the engineer to use mathematics. To do this he must recognize the type of his problem, must understand the method of solving that type, and must understand the construction of the method; just as he must know in any case what tool to use, how to use it, how to modify it, to adapt it or even to make it." The results are reported as "on the whole very good."

(*i*) *Problems from "Real Material."*

Whenever possible real material is used for problems which is obtained from "laboratories, kinematics, graphic statics, etc.—we aim to supplement rather than duplicate the problems in these courses." The following are two illustrations of the problems used:

> "An elliptical ceiling in a church has a 20 foot span and a 6 foot rise at the center. Rafters run tangent to it at angles of 30° and 60° to the horizontal, with a horizontal on top. Find where their intersections are, their lengths, and where the two at 60° strike the wall. Draw same to scale."
>
> For algebra: "The diameter of a solid wooden column is given by the formula $d^2 - 15d\ (F/P - 1) - 700\ (F/P - 1) = 0$; P = ultimate strength (lb./in.^{2}), F = crushing strength (lb./in.^{2}). For white oak F = 5000, P = 4586. Find d."

(*ii*) *Laboratory Work.*

The students make models of the solids mostly used and do a great deal of drawing to scale with pencil, using "cross-section paper, polar coordinate paper ruled both for degrees and radians, logarithmic, sine ruled, etc." They complete about one hundred illustrations of various curves and graphic solutions in this way.

INSTITUTION NO. 11. NATURE OF APPLIED MATHEMATICS.

"The study of Mathematics, as pursued at this school, is chiefly for the purpose of acquiring a working knowledge of its use in the subsequent studies of engineering, physics, and chemistry, and not merely as a component part of a general education." The chief dif-

ference from the usual work given the arts students lies, says one instructor, in "the omission of topics which, we have found, do not make any impression on the student's mind, or add to his skill in analysis and in manipulation, and are of no particular use in his subsequent study. I find it useless to teach such things as undetermined coefficients, partial fractions, permutations and combinations in the Freshman year. I give plenty of time to the calculus and teach the things when they are needed, so that the student may see what they are good for, and have some chance of retaining them."

(*i*) *Graphic Algebra and Trigonometry.*

The freshmen take "Graphic Algebra, Curve Tracing and Algebraic Analysis" for the first three months. Besides the work on graphics the most striking features are "convergency of infinite series; use of infinite series in approximation calculations; errors of observations; methods of least squares." The next three months are devoted to acquiring "a knowledge of the trigonometric functions in analysis and in shortening computations." The remaining three months are devoted to the solution of the oblique triangle, "limits, expansion of functions in series, application of De Moivre's Theorem," and spherical triangles.

(*ii*) *Analytic Geometry and Calculus.*

Analytic Geometry is taken the first three months of the second year. "The matter and methods are intended to aid the student in his subsequent reading of technical literature, and in solving the problems which arise in his work in Mechanics and Physics."—The work in the calculus considered from the engineer's standpoint occupies the remainder of the year and considers differentiation, properties of tangents and normals and problems in maxima and minima.

(*iii*) *Problems from Engineering.*

"No special reference is made to engineering problems in the first year, and it has been considered unprofitable to use material from engineering practice or engineering literature without first carefully preparing it for our students. It is necessary to avoid the appearance of teaching engineering or physics but to make it clear to the student that we are merely teaching the kind of mathematics useful in engineering.—I never got satisfactory results until I cut loose from all text-books, that is, class-room use of text-books."

INSTITUTION NO. 12. MATHEMATICAL POWER AND FORMAL COURSE.

The aims and results of this institution are very well described by an instructor: "The objects I have in view with students are twofold; first, to develop in them mathematical power, including insight and initiative, variety and strength of attack, all dependent of course not upon guess work but upon clear, solid analysis; second, to give them control over the formal part of mathematics as a tool and its methods. The results have been the deepest possible interest in the subject by multitudes of students and a disposition to place mathematics study foremost in the favor and devotion of students."

(*i*) *Problems from Engineering.*

In analytic geometry and the calculus special emphasis is given problems in engineering in order to bring before the student an early realization of the great importance mathematics plays in engineering practice. Cross-section paper is widely used in the freshman work and the students taught to deduce the results directly from the graphs. Texts are made the basis of the work but problems are drawn from a variety of sources.

INSTITUTIONS NOS. 13 AND 14. ENGINEERING PROBLEMS.

In two of the institutions the mathematical studies are taken up with special reference to their connection with mechanics. Problems are selected from engineering data, the students being taught how to obtain and make use of the same successfully together with the use of tables and computing devices.

INSTITUTION NO. 15. EXTRA PROBLEMS FOR ENGINEERS.

This institution adds a sufficient number of problems to the course given to the students in engineering to double the time for the same subject given to the arts students. The bulk of these practical problems relate to surveying while considerable attention is given to the selection of trigonometric equations as a grounding for future analytic work.

INSTITUTION NO. 16. DIVISION INTO TWO CLASSES: ANALYSIS AND COMPUTATION.

Two courses are given simultaneously the first half of the freshman year. The analytic side of algebra, trigonometry and ana-

lytic geometry is considered thoroly in what is designated as "A Course in Analysis." The other is "A Course in Computations;" the various computations arising in connection with work not involving the calculus are taken up and computating instruments such as planimeters, slide rules, etc., are studied and used.

(*i*) *Contents of Course in Computation.*

This work is given in two-hour periods; the first hour being used by the instructor in a lecture, after which the students work at computations under the instructor's direction. The reason for this second course is that it is believed "—that in the elementary branches of the subject the student should be taught 'systematic computation,' which can best be done under personal supervision of the instructor and his assistants."

(*ii*) *Freshman Calculus.*

The second half of the year is devoted to an elementary treatise on differentiation and integration, Ransom's "Freshman Calculus" being used. The purpose of the course is "—to provide the student of science or engineering very early in his course with a familiarity with the fundamental conceptions and methods of the calculus in as far as they are of use in the elementary study of the physical sciences."

INSTITUTION NO. 17. PROBLEMS AND COMPUTATION DEVICES.

The problems thruout the mathematical courses are selected from actual questions arising in connection with "mines, stamp mills, power plants, etc.," in the vicinity of the institution. Permutations and combinations or other subjects of only a disciplinary value are excluded. Tables, slide rules, planimeters, etc., are used in computations.

INSTITUTION NO. 18. COMPUTATIONS.

A short course for attaining accuracy is given in the freshman year. Degrees of accuracy, short methods, use of tables and calculating instruments are the chief features.

INSTITUTION No. 19. COMPUTATION—PURPOSE.

The general idea permeating the work is to insure a working knowledge of mathematics to the students. The particular feature is a course of two hours for three months of the freshman year in mensuration and logarithms, which is a computation course in problems of physics, mechanics and engineering.

INSTITUTION No. 20. LABORATORY WORK.

The special course in mathematics is confined to one two hour laboratory period per week taken in connection with one-half year each of algebra and analytical geometry during the freshman year. A laboratory period is also given to juniors taking analytical mechanics. It has been necessary to discontinue laboratory work in calculus on account of a lack of time.

(*i*) *Instruments and Material Used.*

All work is done in pencil upon "ten by ten to the inch cross-section paper" which is cut to "six and three-fourths by ten, and ten by thirteen and one-half inches." Drawing tables accommodating four students are used.

(*ii*) *Problems Considered.*

"The problems cover the plotting of

$$Y = \sin X, \quad Y = \tan X, \quad Y = \sec X,$$

or the co-functions; which, together with plotting in polar coordinates, gives some review and exercise in trigonometry. In connection with algebra, problems in graphical solution of single equations and pairs of simultaneous equations are solved. Along with analytic geometry are given further exercises in curve plotting, by points, illustrating among other things, inflections (not the actual location), asymptotes, nodes, and cusps, and to some extent the relation between form of equation and form of curve (this being a difficult matter with the student). We also plot certain systems of conics. During the differential calculus I have had series plotted, as:

$$Y = X, \quad Y = X - \frac{X^3}{\lfloor 3}, \quad Y = X - \frac{X^3}{\lfloor 3} + \frac{X^5}{\lfloor 5}, \quad Y = \sin X,$$

illustrating convergency. Another style of problem has to do with curvature; as to draw a parabola and the circle of curvature for the vertex, and then draw the circle of curvature for some other point.

Examination of such drawings, carefully made by the student, is quite instructive. Another exercise is to plot $Y = f(X)$ and several of its derivatives."

(*iii*) *Workings of the Course.*

The instructor in charge of each laboratory section gives it his personal attention. He also "—makes up the problems, all members of the class working on the same problem at about the same time. This of course results in some little copying, but on the other hand, the advantages of comparison, one with the other and the suggestions one student gets from another outweigh that." For calculus they use the "laboratory period as a practice period in the integral calculus, beginning the subject before finishing the differential, the students working at the board, with the instructor passing around answering questions, offering suggestions and correcting the work. In the laboratory period the fundamental formulae were gotten thru with by the time of finishing differential calculus, but the laboratory period was retained thru the integral calculus."

(*iv*) *Object of Course.*

The object is "—to supplement and broaden the analytical part of trigonometry, advanced algebra and analytics. It has been my experience in assigning curves to be plotted at home to have all degrees of accuracy and slovenly work presented and very little good work. The student also appears to regard the graph as an unnecessary picture or carricature of the equation, a thing to be drawn in free hand by an artist of the impressionist school." The poor preparation of the student is assigned as another reason for giving this work, because the personal supervision of the instructor for a long period is thought to be beneficial.

(*v*) *Results.*

As to results and working of the course we read: "Our classes are small, about twenty for freshmen and ten for sophomore mathematics. This makes it possible to provide tables and blackboard room for working a whole class at once. Of course the laboratory work takes more time of the instructor and we have also found another difficulty; that of finding sufficient time during the day for students to take all their necessary laboratory work."

The only objections raised were those of extra demands upon

the teaching force and some difficulty in arranging the schedule of the students. After a four years' trial the plan was reported a success and well liked by the students. It was further said to aid "—in giving not a little practice in numerical calculations and for this we encourage the use of the tables, as of squares and cubes. This is made the occasion of mentioning the slide rule."

V. SUMMARY.

1) Very few institutions mentioned the date at which they had first taken up this modified work in mathematics. No date earlier than 1900 was given.
2) Number of institutions that give trigonometry 16
3) Number of institutions that give algebra ..... all
4) Number of institutions that give plane analytical geometry ... all
5) Number of institutions that give solid analytic geometry ....
   first year 7
   second year 10
6) Number of institutions that give the calculus first year 7
   second year 15
7) Number of institutions that give the calculus as a separate course 14
8) Number of institutions that give the calculus in connection with other work 8
9) Number of institutions that give courses more or less of a laboratory nature 9
10) Material Used:
   i) General text-books used only..... 4
   ii) Text-books arranged for engineering students..... 3
   iii) Specially prepared pamphlets, with or without texts.. 2
   iv) Text-books supplemented by problems and exercises taken from laboratories and professional departments 4
11) Main Features.

Problems given much more prominence than in the mathematical courses for arts students.

Omission of useless and uninteresting topics in algebra—the use of especially prepared lists of problems in algebra.

Computation courses under supervision.

Laboratory periods for work in graphs.

Topics arranged so as to make one coherent course without division into subjects.

Aims to give an early working knowledge of mathematics, especially of the calculus.

Selection of topics which will form a direct part of the future work in engineering subjects.

12) Causes for the Establishment of These Modified Courses.

Good preparation of entering students,—a cause for giving more advanced work.

Poor preparation of entering students,—a cause for giving laboratory work.

Agitation in the journals and at the meetings of the various societies.

To attain the requirements of the professional departments.

13) Objects of the Course.

To create interest in algebra.

To ground students in the solution of problems.

To teach the value of the graph and its use in the interpretation of results.

To introduce the calculus early so as to equip the students as soon as possible with this weapon, and at the same time giving them more practice in its use and for a longer time.

To give a working knowledge of mathematics and ability to use it.

To familiarize the students with the mathematics they will need for their future technical work.

14) Results.

Only a few of the letters received gave any expression as to the results attained in these modified courses. All that did so, however, were highly in favor of the new mode of procedure. Only one instance was found of a return to the more formal treatment. That was the omission of laboratory work in the calculus, necessitated by a lack of time.

CHAPTER IV.

CURRENT THOUGHTS ON VITAL QUESTIONS.

I. MODE OF OBTAINING THE DATA.

To obtain a survey of current thought on certain important questions 651 copies of the following questionaire letter were sent out.

For convenience the questions were, as far as possible, made answerable by yes or no, or by numerical annotation, even at the cost of making them more narrow and formal than would otherwise have been desirable. In order not to make (2), (4) and (7) complete tables of contents for these various subjects and thus render them too large to be readily answered, topics *a* under each had to include a larger number of operations than would otherwise have been desired.

1) QUESTIONAIRE LETTER.

UNLESS OTHERWISE STATED THE FOLLOWING WILL RELATE TO THE COURSES IN MATHEMATICS FOR FRESHMEN STUDENTS IN ENGINEERING.

1. Is it advisable to require entrance examinations in mathematics of all students?

2. Number 1, 2, 3, etc., in order of importance the following topics which are fundamental to a course in trigonometry:

- *a. Derivation and manipulation of formulae.*
- *b. Solution of triangles by use of natural functions.*
- *c. Use of logarithms in solution of triangles and other computations.*
- *d. Problems applying the solution of triangles.*
- *e. Higher trigonometry—hyperbolic functions, DeMoivre's Theorem, etc.*

3. In what particular topics of trigonometry do students show themselves most deficient after the freshman year?

4. Number 1, 2, 3, etc., in order of importance the following topics which are fundamental to a course in college algebra:

a. Review of surds, exponents, quadratics, etc.
b. Series.
c. Binomial Theorem.
d. Permutations and combinations.
e. Graphs.
f. Determinants.
g. Theory of Equations.

5. In what particular topics of algebra do students show themselves most deficient after the freshman year?

6. Should the object of analytic geometry be (a) to learn the characteristics and theory of some class of curves, as conics, or (b) to learn a new mathematical language, using any suitable problems in curves or mechanics as the medium for the study of this language?

7. Number 1, 2, 3, etc., in order of importance the following topics which are fundamental to a course in analytic geometry:

a. Algebraic relations for intersection of loci, tangency, etc.
b. Loci problems.
c. Right line and circle.
d. Conics.
e. Higher curves, as cycloids, etc.
f. Coordinates of points in space with application to simple loci.

8. In what particular topic of analytic geometry do stdents who have completed a course in that subject show themselves most deficient?

9. Would it be advisable to introduce a working knowledge of elementary differentiation and integration preceding or simultaneously with the course in analytic geometry?

10. What is the remedy for (3), (5), and (8)?

11. Is it advisable to treat problems met with in actual engineering work?

12. If so, to what extent and under which topics of (2), (4), and (7)?

13. Is a short course in computation advisable?

14. Should freshmen engineers be taught their mathematics in separate classes or together with those taking other courses?

15. Why?

16. Are special texts in mathematics for engineering students advisable?

17. How often should written quizzes be given?

18. Number 1, 2, 3, etc., the order of usefulness of the following modes of conducting classes suggesting any further modes or combinations of these:

- *a. Assignment of work from text without any previous explanation.*
- *b. Assignment of work from text with previous explanation.*
- *c. New work explained, the student taking notes—with or without the use of a text. Written or oral quizzes at regular intervals.*
- *d. Students led to discover new principles by suggestions and quizzes on the old.*
- *e. Each student working independently, going as fast as he is able. This may apply to the entire course, to each topic, or to each day's work.*
- *f. First part of hour devoted to the class as a whole; second part given to individual work.*
- *g. Subject treated from the laboratory standpoint; most of the work being done in the class room.*

19. Number 1, 2, 3, etc., the order of importance of the following pertaining to the qualifications of the engineering student who has completed the courses in freshman mathematics:

- *a. Skill and accuracy in computations.*
- *b. Analysis of problems.*
- *c. Interpretations of results in solution of problems.*
- *d. Knowledge and use of equations.*
- *e. Representation of physical laws by means of graphs.*

20. In which of (19) do students who have completed their freshman year show the greatest deficiency?

21. What is the remedy?

22. Designate by 1, 2, 3, etc., the order of importance for any mathematical topic of:

a. Mastery of theoretical matter involved.

b. System and neatness.

c. Accuracy in computed results.

23. Is it true, as has been said, that engineering students apply themselves better than those taking a literary or scientific course?

2) REPLIES.

The six hundred and fifty-one letters sent and the replies received are classified in the following table:

	No. Sent	No. of Replies	% of Replies
To instructors of mathematics and professional courses, representing 92 institutions	544	204	37.5
To teachers of mathematics especially interested in the question	20	13	65.0
To professional engineers	87	20	23.0
Total	651	237	36.4

The practical side is represented more strongly in the replies than the number would indicate, since several of the instructors of professional subjects are also practicing engineers.

II. TABULATION OF DATA.

A summary of the replies to these questions is given in what follows. The questions are taken up in order and separately. Following a short review of the important points considered in the question are appended some of the most suggestive and helpful comments in the form of quotations. The discrepancy which will often be found between the total number of replies received and that recorded for any particular question is accounted for by the fact that several writers omitted replies to various questions.

III. ENTRANCE EXAMINATIONS.

1. *Is it advisable to require entrance examinations in mathematics of all students?*

Replies:

Yes	130
No	72

Several of those who opposed this requirement did so on the ground of inexpediency but would otherwise have favored it, while many who favored it voiced the same objections; namely, (a) that it would cause serious disturbance in the accredited secondary schools and (b) that if the rule of an institution is to receive its students from the secondary schools without examination it would not be fair to the other departments to make an exception in favor of mathematics.

Few complaints were found of poor preparation in geometry but a very large number in algebra. The general opinion seemed to be that something stronger than a mere request should be presented to the secondary schools for better preparation in algebra. The following quotation expresses the reasons in favor of entrance examinations:— "I think that all students entering engineering courses should be required to take entrance examinations for the following reasons: 1st, to find their deficiencies in order that these may be remedied before it is too late; 2nd, to insure that the high school preparation is what it should be; 3rd, to insure that a good preparation has not been forgotten; 4th, to discourage men deficient in mathematical ability from undertaking the engineering courses without extra preparation, and to prevent those lacking in such ability from entering such a course at all."

1) A Substitute for the Entrance Examination.

One method which is, in a way, a compromise between these two extremes is used by some of our foremost and rising institutions. This plan, as outlined in a letter from one of the institutions using it, is as follows: "Regarding Entrance Examinations in Mathematics,

I would say that it seems to us not desirable to require Entrance Examinations in the case of a State University, even in such a subject as Mathematics, on account of the fact that the University should take its place as part of the general school system of the state. This it cannot do if it stands aloof from these schools, and accepts their students only after an examination, which explicitly refuses recognition of the quality of the work done in the secondary schools.

"We recognize, however, that the students entering on certification are often very deficient in mathematical knowledge. In order to avoid the setting of examinations, and to provide some basis for weeding out the poorest of these students, we have struck upon the following scheme: Some two weeks or more after the opening of school, after a rapid review of those topics in elementary algebra which precede quadratic equations, we give a test covering this ground. The students who fail in this test are not allowed to continue this course. We do not remove their credits for entrance, we do not even require them to take up the study of elementary algebra, but we make it rather obvious that the only way they will ever succeed in this course is by a thoro review of that work, and we point out to them that an opportunity to do this is afforded in several of the Secondary Schools in operation in this city. To the next grade of students, those who do not utterly fail, but are weak on this test, we give the alternative namely, that they either drop the course at once, or that they continue it, together with a review of elementary algebra, in the training school of the Teachers' College of his University. We believe that this scheme is superior to the scheme of entrance examinations, and highly superior to the scheme of direct admission to classes on certification without restriction."

No unfavorable reply was received from any one who had ever tried this plan.

IV. SPECIFIC NEEDS AND DEFICIENCIES.

The following seven questions are concerned with the needs, deficiencies and remedies pertaining to topics in the subjects generally taught the freshman year; logarithms, trigonometry, algebra, determinants, etc.

1 TRIGONOMETRY—*i) Needs.*

2. *Number* 1, 2, 3, *etc., in order of importance the following topics which are fundamental to a course in trigonometry:*

a. Derivation and manipulation of formulae.
b. Solution of triangles by use of natural functions.
c. Use of logarithms in the solution of triangles and in other computations.
d. Problems applying the solution of triangles.
e. Higher Trigonometry—hyperbolic functions, DeMoivre's Theorem, etc.

Replies:	I.	II.	III.	IV.	V.	F.
a	147	17	21	34	1	935
b	40	63	37	63	6	695
c	33	92	54	29	1	754
d	33	40	79	55	4	676
e	0	0	9	11	158	207

The columns of the above table, indicated by the Roman numerals, refer to the order of importance assigned to the various topics mentioned in the question. The number in any column opposite any letter tells how many of those answering the question gave it this particular order of importance. Thus there were 37 who placed *b* as third in order, 11 who placed *e* as fourth, etc. The numbers in the final column, marked F, are obtained by adding 5 times the number in the first column, 4 times that in the second, etc., as for *a* $147\times5 + 17\times4 + 21\times3 + 34\times2 + 1\times1 = 935$, which shows the relative importance assigned to *a, b, c, d, e.* It should be mentioned, however that a large number thought *e* should be wholly omitted.

Comments:

"In trigonometry, more time should be devoted to practical problems in plane trigonometry; much less to spherical trigonometry, which is rarely used even by civil engineers. It is singular that perhaps more students fail in trigonometry than in other branches; as it is comparatively easy, the system of instruction must be at fault."

"Few students are drilled to use logarithms intelligently, rapidly or correctly. Most tables of logarithms and logarithmic functions in American text-books are very inconvenient and far inferior to the German tables in

arrangement, and instructors seem scarcely to consider this in selecting a text-book. Hence the student never likes to use logarithms. Then he is often required to use 6 or 7 place tables for computation in which 4 place tables would be amply sufficient."

"Regarding the importance of the topics which you mentioned in trigonometry I would say that the order appears to me to be as follows: *b, c, d, a.* I have omitted *e* entirely, for its importance is highly problematical in a course on trigonometry for freshmen, since I do not believe that freshmen have any use for this matter, and I do not believe that they can get a proper grasp of it. I have placed *b* first simply because it seems to me to involve the true spirit of trigonometry as a whole. If the student grasp the essential notion of the solution of right triangles, he has a foundation for the whole of trigonometry. I place *c* next since the question of computation by logarithms and otherwise is the main use to which the freshman will put his trigonometrical knowledge. I place *d* next since the practical problems of trigonometry are the type to which he will apply this knowledge. The derivative and the manipulation of formulae are important enough, but they are certainly secondary to the general grasp of the purpose of elementary trigonometry, which is essentially contained in the first three.

"The spirit of the remarks I have just made will follow through all I have to say in what follows, namely, in each subject. I shall regard as of first importance the general conception of that subject, next the particular means of putting these into operation, and next the things to which they are applied."

(*ii*) DEFICIENCIES.

3. *In what topics of the above in trigonometry do students show themselves most deficient after the freshman year?*

Replies:	*a*	*b*	*c*	*d*	*e*
	126	22	33	34	12

Very few replies reported difficulties under more than one topic given in question (2). It is interesting to note that 55% of the difficulties reported fall under *a*, the derivation and manipulation of formulas. A few of the letters specified particular topics under *a* as giving the greatest amount of difficulty as follows:

1) 50% in the manipulation of formulae, especially in identities, functions of 2x and ½x;
2) 25% in inverse functions;
3) 12½% in circular functions;
4) 12½% in application of definitions.

Comments:

"In trigonometric equations."

"They can not apply definitions to rapid solution of triangles."

"The greatest weakness I find in the student having finished trigonometry is an inability to interpret a given problem in the trigonometrical terms; that is, he is unable to see the triangles which must be solved."

"Technical engineering courses (undergraduate) require, as a rule, little mathematics beyond the elements. Hence in these courses it is only in the elements of mathematics that students show either weakness or strength. When the attempt is made in the freshman mathematics courses to cover much ground including 'advanced' work, the inevitable result with a large proportion of students is weakness in the elements, which reveals itself in all subsequent work involving mathematics. The remedy, as far as a remedy is possible, is to confine the work in the freshman and sophomore mathematical subjects mainly to the elements and to teach these as efficiently as possible."

2) ALGEBRA (*i*) *Fundamentals.*

4. *Number* 1, 2, 3, *etc., in order of importance the following which are fundamental to a course in college algebra:*

a. Review of surds, exponents, quadratics, etc.
b. Series.
c. Binomial Theorem.
d. Permutations and combinations.
e. Graphs.
f. Determinants.
g. Theory of Equations.

Replies:	I.	II.	III.	IV.	V.	VI.	VII	F.
a	177	15	11	3	1	0	0	1399
b	2	34	42	51	26	15	10	639
c	19	83	56	19	15	5	0	1032
d	1	4	14	34	36	36	48	465
e	23	52	48	33	24	8	10	895
f	9	4	14	14	38	60	38	426
g	9	25	24	27	26	34	28	615

The key for (2) also applies to the above table. A few replies gave an equal rank to two or more topics, others omitted one or more as non-essential to engineering students. Of the latter *f*, *c*, and *d* were the principal ones mentioned and in that order.

Comments:

"I should omit the subject of determinants and give very little time to permutations and combinations on a course in engineering algebra. Graphic analysis should be introduced in the high school and should be taken up in connection with all of the topics studied.

"In addition to the subjects mentioned under (*4*) a thorough course in Limits and Logarithms is essential to the work in algebra. I believe I would class these two subjects as of greater importance than the Binomial Theorem."

"A good deal of the work which we often attempt to do in advanced algebra is better done after a working knowledge of analytics and calculus has been obtained, and work taught without such interrelation is both more difficult to master, and without its full value. For the purpose of engineering education, a thorough grounding in the simpler elements of higher mathematics is immensely more desirable than a perfunctory acquaintance with the frills of mathematical theory, such as some of the higher curves having remarkable mathematical properties. Curves having remarkable properties of engineering value, as for instance the involute, cycloid, logarithmic spiral, etc., should be considered, as should also the properties of exponential curves, and the use of logarithmic cross-section paper."

"During the college course in algebra an attempt is generally made to cover too much ground rather than to do the work thoroughly. On account of algebra being often poorly taught in high schools by the weakest teacher, like English grammar, many students fail in this course."

"Few students are able to transform a given algebraic formula, insert numerical constants, placing it in the form most convenient for practical use, for example, in case of formulas relating to the safe strength of materials."

In brief, upon a defective foundation in arithmetic and algebra, the professional mathematician endeavors to erect a superstructure of the higher mathematics, which in most cases never becomes a permanent part of the student's mental equipment and is therefore rarely utilized and is forgotten quickly after entering on the practice of his profession."

(*ii*) *Deficiencies.*

5. *In what particular topics of algebra do students show themselves most deficient after the freshman year?*

Replies:	*a*	*b*	*c*	*d*	*e*	*f*	*g*
	114	24	17	8	14	9	41

A few replies mentioned difficulties in more than one topic of algebra. It appears at once that topic *a*, review of surds, exponents, quadratics, etc., covers 50% of all the trouble recorded. In the replies which gave specific details regarding *a*, surds were mentioned the most frequently; and then exponents, quadratics, imaginaries and manipulation of algebraic symbols in about the order given.

Comments:

"In simple algebraic reductions, especially where radicals are involved."

"Fundamental operations; such blunders as $1/a \div \sqrt{b} = \sqrt{ab}$."

"In everything beyond quardatics."

"Solution of n linear equations with n unknowns and of equations where exponential functions are involved."

"They show a general want of freedom owing to undigested loads of information."

"Inability to set up equations applying to known or observed facts; more practice should be given along that line."

"My students in electrical engineering are invariably deficient in their algebraic training regarding the uses and powers of imaginary quantities. These factors and the physical meaning attached to them are of great importance in the treatment of alternating currents. I find, as everywhere, the students are deficient in algebraic power through lack of experience rather than lack of extent of algebraic training."

3) ANALYTIC GEOMETRY—(*i*) *Object.*

6. *Should the object of analytic geometry be* (*a*) *to learn the characteristics and theory of some class of curves, as conics, or,* (*b*) *to learn a new mathematical language, using any suitable problems in curves or mechanics as the medium for the study of this new language?*

Replies:	*a*	*b*
	56	172

It appears then that a trifle over 75% of the replies to this question favored *b*, the learning of a new language as the object of analytic geometry.

Comments:

"Very decidely (*b*). Students generally fail to get a definite grasp of the relation between the number of conditions a curve can satisfy and the corresponding number of constants in the equation of the curve. Some exponential and logarithmic curves should be used."

"Most decidedly (*b*) is the important element here, that is, to learn a new mathematical language, using any suitable problems in curves or mechanics as a medium for the study of this new language; and to become thoroughly familiar with it and to acquire power in its application."

"Regarding the alternative which you offered between (*a*) and (*b*), I should be obliged to say (*b*), but I would rather rephrase the matter myself thus: I do not believe that the course in analytic geometry should be

in any sense a treatment of Conic Sections, nor that it should attempt to bring out merely the geometrical properties of any certain class of curves; rather the whole object should be to familiarize the student with the notion of representations of equations by graphical figures, and a representation by figures of functions divorced entirely from the equation idea, together with as great a familiarity with the simpler forms of curves as a thorough treatment can give, so that he retains at least a rough knowledge of the appearance of the curves in question; I mean forms of the class:

$$ax + b,\ ax^2 + bx + c,\ kx^n,\ (ax + b)/(cx + d),\ \sin x,\ \cos x,\ e^x,\ \log x,$$

and other simpler forms, with perhaps some knowledge of the general equation of the second degree, certainly with a knowledge of such forms as $x^2/a^2 + y^2/b^2 = 1$, etc.; the whole spirit of a class in analytic geometry should be to familiarize the student with this possibility of geometric helps in algebraic problems and the representation of functions."

(ii) Importance of Topics.

7. *Number* 1, 2, 3, *etc., in order of importance the following topics which are fundamental to a course in analytic geometry*:

a. Algebraic relations for intersection of loci, tangency, etc.
b. Loci problems.
c. Right line, and circle.
d. Conics.
e. Higher curves, as cycloids, etc.
f. Coordinates of points in space with application to simple loci.

Replies:	I.	II.	III.	IV.	V.	VI.	F.
a	115	31	21	10	5	4	973
b	40	77	34	32	7	0	880
c	51	54	74	9	1	4	905
d	11	20	39	88	22	2	622
e	11	5	16	16	76	52	407
f	10	13	8	21	49	81	399

The key for the above table is the same as for question (2). This question is very closely related to question (6) which should be thought of in connection with it. Nearly all the answers were in terms of *a*, *b*, *c*, etc., or such as to be directly translatable into these.

Comments:

"Lacks ability to set up problems in algebraic symbols."

"No special emphasis should be given conics. They should be regarded merely as one class in the discussion of curves in general together with trigonometric, logarithmic, algebraic and transcendental curves."

(*iii*) *Deficiencies.*

8. *In what particular topic in analytic geometry do students who have completed the course in that subject show themselves most deficient?*

Replies:	*a*	*b*	*c*	*d*	*e*	*f*
	47	61	8	25	13	20

In a few instances this was not answered in *a, b, c,* etc., but in statements which had to be interpreted into that of topics in question (7). Two chief difficulties were mentioned; the inability of students to correlate algebraic and geometrical ideas and to sketch in curves from the inspection of their equation without going into the detail of plotting them.

Comments:

"The ability to sketch the locus from an inspection of the equations. Determining the nature of a locus from its equation and interpretation of results."

"In expressing laws in algebraic language and deducing the general form of the locus from the equation."

"A general lack of understanding the usefulness of the subject and hence superficial knowledge of the fundamentals."

"In my work they usually show little knowledge of such curves as

$$y = ax^n + b;$$
$$y = ae^x;$$
$$y = a \sin (x + b).$$

"After having had analytic geometry the students show themselves deficient in two ways in their calculus. Their deficiency is largely failure to grasp the general notion of plotting a function, and also failure to appreciate the tangent problem. In their later engineering work their deficiency in analytic geometry is largely on the side of empirical curves, and obtaining of equations from empirical data. In this they are scarcely to be blamed, for this is a new notation which can at the best be illustrated by lame examples in their first course in analytic geometry."

(*iv*). *Derivatives and Integrals in Analytic Geometry.*

9. Would it be advisable to introduce a working knowledge of elementary differentiation and integration preceding or simultaneously with the course in analytic geometry?

Replies: Yes. (together).....115
No. 71

In addition to the considerable majority of opinion in favor of the early introduction of the elements of the calculus (62% of the total number of the replies) the striking fact was also brought out, that all who reported having tried this method were, without exception, in favor of it and had found it a success. On the other hand several who favored the general proposition limited it to the introduction of differentiation only.

Comments:

"No, haven't time."
"Should begin this early in algebra."
"I have combined both for the last ten years."
"Yes. I carry engineers thru analytics and calculus simultaneously."
"I think it highly advisable to treat the tangent problems by this method and to give the student a working knowledge of the simpler processes of differentiation along with their analytic geometry."

4) AIDS AND REMEDIES FOR ABOVE.

10. What is the remedy for (3), (5), and (8)?

The replies to this question cannot be exhibited in the tabular form previously used. The remedy given most frequently was to *apply the theory to practical problems.* Two reasons were found for this; first that the greatest difficulty met by students later on was in the application of principles which *had been learned,* and second, that this would create interest and realization of the importance of the topics studied. Another remedy frequently proposed was "more drill, more drill" and in direct connection with this the plea for frequent reviews.

Comments:

"Review constantly."

"Special short course sophomore year."

"Less extension and more intension in study and drill."

"Make necessary topics more important, other less so."

"Give simple problems illustrating laws of mechanics and physics."

"Smaller number of students; copious problems with engineering application; instructors who know needs of engineer."

"No one thing. Students would do well to think more and hurry less. Perhaps it would be well if writers spent more time in pointing out errors that have been made and are to be guarded against."

"To extend the time devoted to the subject so as to admit of a more extensive drill, and to draw upon engineering practice for concrete problems which will arouse the student's interest and hold his attention."

"Greater accuracy should characterize the high school and grades. The habit of accuracy can not be developed in a day or in a year."

"Emphasize mathematics as a tool and have it thoroughly mastered from that standpoint."

"More drill; selection of problems they will meet in second year and lay stress upon that fact."

"Cut out at least one-third of the theoretical stuff and use the time given to it in practical arithmetic and graphical problems in engineering."

"Working many problems taken as far as possible from practical applications either real or possible. The practical application awakens interest, and interest overcomes many difficulties. We should in all cases show the relation as noted in your section (*6b*). The problems should be carefully corrected, not merely marked or other methods of solution shown, but the student's fallacies or errors pointed out. There should be frequent quizzes or drills. I prefer to devote the *first* half of every period to drill and the last half to explanations of next lesson and clearing up any difficulties in quiz just given. The quizzes should overlap somewhat; i. e. should include work done in the last quiz or two, or there should be a general quiz every 5th period."

"Too much stuffing and learning by rote."

"Early training to be analogous to that in English and German."

"Redistribution of emphasis in teaching."

"A logical course in mathematics extending three years, not algebra, trigonometry, etc., as usually taught."

"Vitalizing mathematical work by giving more time to its concrete application in physics, chemistry and engineering and less to mathematical training seldom of use in practice."

"I have noticed no predominating deficiency. The general deficiency is failure to recall and apply the knowledge attained earlier. To forestall

this as far as posible it would seem advisable that the Freshman instructor make clear the purpose of the subject. Make the few fundamental principles and methods stand out boldly. Aim to cultivate the mathematical type of thought. Show how the few basic methods of each subject act in slightly different varying ways to solve many important problems and give considerable practice in actually doing this."

"The subjects of algebra, trigonometry and analytics should be less rigorously separated into water-tight compartments. The solution of simple plane triangles should be brought in with plane geometry, using only natural functions of, say, acute angles. Considerable trigonometry can be brought out in analytic geometry and still more in calculus. Algebra and trigonometry can be united in work in series to get roots of imaginary complex quantities by DeMoivre's Theorem. Whenever a new method has been developed, the teachers should point out and emphasise the practical problems and examinations which *do not presume* that a grade in a subject makes continued knowledge in that subject necessary."

"The remedy for questions (*3*), (*5*), and (*8*) in my judgment is presistent practice required of the students in application of principle to practical problems. The use in explanation by the instructor of the simplest non-technical language and abundant reference to the homeliest illustrations at his command. I believe that the crucial difficulty in the case of the average student of mathematics is his fear of the subject born largely of a too generous use by instructors of the technical and unfamiliar language, and the failure to use wherever possible, illustrations drawn from things with which the student is commonly familiar. We also correlate as thoroughly as may be, the subjects which are allied to mathematics with the mathematical instruction itself; as for instance, applied mechanics, physics and applied electricity."

"For question (*8*) the tracing of all curves met with further on."

"Some algebra should be taught every year in the high school."

"Make trigonometry more *simple*. Use more coordinate and graphic methods."

"Connect analytic geometry with calculus."

"Drill on (*a*) in each with special attention to clear statements and full complete reasoning."

"For questions (*3*) and (*8*) give greater emphasis on algebraic notation as shorthand translation for English into algebra and vice versa."

"At the beginning of analytic geometry a short and thoro drill on the kind of algebraic process and trigonometric relations to be used is advised."

"A similar drill just before starting calculus on some of the most needed algebraic, trigonometric and analytic relations would largely remedy the difficulty. For example, a sharp drill on a few of the most fundamental relations of trigonometry would help in differentiation and integration of trigonometric forms."

V. MODIFICATIONS IN MATHEMATICS FOR ENGINEERS.

1) ENGINEERING PROBLEMS—(*i*) *Advisability.*

11. *Is it advisable to treat problems met in actual engineering work?*

Replies: Yes 190
No. 20

The answers to this question while overwhelmingly in the affirmative—a little over 90%—contained many warnings against overdoing and the use of too technical material. They also strongly condemned the use of such problems by instructors ignorant of the physical meaning of the quantities involved. The principal objections were the lack of time and the lack of such problems.

Comments:

"Very seldom. We want principles and not applications."

"No, because such problems have too many unknown quantities in them."

"No, it is better to use ideal problems of physics, chemistry, etc."

"Yes, but there are objections:—Not sufficient time. No such problems perhaps at hand. Difficult to get problems of requisite simplicity and near enough to actual engineering problems to be of any value."

"Yes, without duplicating teaching."

"Yes, adds interest but they must necessarily be elementary."

"Simple problems. Do not make the common mistake of using applications which are Greek to the students."

"Yes, thorough drill in problems met with in land surveying."

"Regarding the problems from actual engineering work, I would say that I should favor very strongly the introduction of such problems, provided they are so selected as to be within the reach of the student's present knowledge. They should not be absurd, as are some problems; *i. e.*, they should correspond to some real facts, and they should not be fake practical problems. They should not be a statement of such a somebody's formula, where the various letters mean certain unheard-of things. These restrictions limit the possible material enormously, but I am convinced that they should be insisted upon."

"Yes, to bring teachers of mathematics into close touch with teachers of engineering."

(*ii*). *Nature of Such Problems.*

12. If so, to what extent and under which topics of (*2*), (*4*) *and* (*7*)*?*

Replies:

For (*2*)

a. Derivation and manipulation of formulae 11
b. Solution of triangles by use of natural function 19
c. Use of logarithms in solution of triangles and other computations 35
d. Problems applying the solution of triangles 39

For (*4*)

a. Review of surds, exponents, quadratics, etc. 20
b. Series 19
c. Binomial Theorem 13
d. Permutations and combinations 4
e. Graphs 26
f. Determinants 1
g. Theory of equations 12

For (*7*)

a. Algebraic relations for intersection of loci, tangency, etc. 18
b. Loci problems 28
c. Right line, and circle 23
d. Conics 17
e. Higher curves, as cycloids etc. 7
f. Coordinates of points in space with application to simple loci 7

The answers which referred to explicit topics in questions (*2*), (*4*), and (*7*) are embodied in the above table. This means that thirty-nine replies suggested *d* for question (*2*). *d* was not the only suggestion in these thirty-nine replies, however, as several contained two or three. While the number of answers received is not so very large the opinions expressed are quite uniform, as will be seen by referring to the above table.

Comments:

"To limited extent only under any topic needed in solution."

"Only a few problems. The tendency in modern text-books, is to treat too many problems of the kind."

"In all, mainly in (*2*) and (*7*)."

"Graphic solutions wherever possible."

"Follow a principle with an illustration."

"Whenever possible. They should require little technical knowledge. It is easy to over do this thing."

"Problems applying solution of triangles, series, graphs, conics and higher curves."

"Paths for (*7*). Polygon of forces for (*4*) and (*2*), (*2*)*c* resolution of forces, bridge structures, etc., (*7*)*c* gear teeth, (*7*)*f* intersections of solids, with models."

"(*2*)*b, c, d* surveying and railroad curves."

"To awaken interest and show relation between abstract and applied mathematics:—(*2*) *b, d;* (*4*) *c, f, g, d;* (*7*). Call attention to their appreciation rather than problems."

"The engineering problems should come as exercises of the application of mathematical principles, rather than as a means of teaching those principles. These illustrative problems should be introduced in connection with every topic possible. I believe that the reason so much of the mathematics studied by the engineering students falls into disuse and becomes forgotten, is that the average student has to be shown how to use practically what has been studied theoretically, and if not shown, will never get the practical use out of what he has learned. The mind seems to be so occupied in grasping the mathematics involved, that the application is lost unless specifically illustrated."

2) Advisability of a Course in Computation.

13. Is a short course in computation advisable?

Replies:	Yes	124
	No	49

Several who favored such a course added either qualifying statement, "if there is time for it," or "if there is need for it." Many who opposed such a course were really in favor of the plan except for time it would require. In one quotation under question 20 an important point is brought out; namely, that while students are continually admonished to be accurate, they are, as a rule, told very little about the degree of accuracy required by the various problems encountered and given next to no practice in the same.

Comments:

"Good, but no time for it."

"Should be emphasized in all courses but not given as a separate course."

"A large number of computation problems should be given in connection with the different courses. I would not have separate computation courses."

"Review."

"By all means, a short course, or a course to be made as long as possible."

"Yes, particularly if supplemented by graphical methods; i. e., use of slide rules."

3) SEGREGATION OF ENGINEERS IN MATHEMATICS—(*i*) *Advisability.*

14. Should freshmen engineers be taught their mathematics in separate classes or together with those taking other courses?

Replies:	Separate	136
	Together	55

71% of the opinions expressed were thus in favor of the special sections for freshmen students in engineering. A few who voted "together" did so because the practical training needed by engineers would also be the best for the literary and general science students. The reasons given are taken up under the next question.

(*ii*) *Reasons.*

15. Why?

In the following quotations those under (a) state reasons why separate classes are unnecessary, those under (b) why they are necessary, and those under (c) why the literary and general science students should take their mathematics in classes organized for the engineering students.

Comments:

(a) "There is no engineering mathematics."

"Good mathematics for one is good mathematics for the other."

"Aids in cultivating wider social sympathy."

"In order to not lose its culture value apart from application."

"Because mathematics should be studied as a science rather than as a mere working tool for engineers to be applied mechanically."

(b) "So as to emphasise engineering application."

"Mathematics for engineers has two objects: the development of the student and the practical value."

"The engineers aim at facility in computation and the use of formulas, while the art students go more into foundation and theory."

"Because the men in other courses will not probably appreciate the spirit in which the subject should be attacked. Not because the method or course should be different."

"Mathematical rigor need not be insisted upon. Either to select illustrative material which will apply to whole class."

"Realization of the greater certainty of the practical use made of their mathematics will probably result in the average engineering student accomplishing more than will be accomplished by the non-engineering student, if the former is unhampered by the company of the latter."

"Because the instruction for the freshman is the most important instruction, especially in mathematics during the four years' course. The freshmen require a very specific and individual treatment as far as possible. It seems so fundamental that they should be taught right ways of thinking as well as right mathematical facts, that they ought to be a special and entirely separate problem."

(c) "What is good for the engineer will be good for others, even the practical problems."

"The emphasis of the practical does not detract from the educational nature of the subject."

4) Special Texts.

16. Are special texts in mathematics for engineering students advisable?

Replies: Yes 93
No 89

Nothing of importance was brought out by this question other than the results given above and the quotations which are appended.

Comments:

"They are not as a rule satisfactory."

"Rather special teachers."

"Only if it is written more clearly."

"Not necessary. Supplementary material should be chosen with reference to students."

"Such texts may save the students valuable time."

"No, all texts should be given a turn toward engineering."

"Perhaps for analytics and calculus but not for algebra."

"Only for analytic geometry where there is room for a real working text."

"Yes, if the teacher can not supply what is lacking in the theoretical text-book."

"Yes and no. Theoretically it would, but I know of no such texts."

VI. Pedagogic Questions.

1) Quizzes.

17. How often should written quizzes be given?

Of the 181 formal answers received 32 thought quizzes should be given monthly, 10 every there weeks, 55 every two weeks, 22 weekly, 25 frequently, 1 daily, 1 seldom, and 35 at the option of the instructor. The general expression was, of course, that no fixed rule could be given.

Comments:

"As often as instructor can stand them. Should be left entirely to him."

"Frequently, with enough time given to allow the students to think out the problems."

"In my work I find it so full of interest to the pupils and the time at my disposal so limited that I dread to lose good time for the purpose of a quiz. Every quiz means a lot of good information lost to the class."

2) Modes of Conducting Classes.

18. Number 1, 2, 3, etc., the order of usefulness of the following modes of conducting classes, suggesting any further modes or combinations of these:

a. Assignment of work from text without any previous explanation.

b. Assignment of work from text with previous explanation.

c. New work explained, the student taking notes—with or without the use of a text. Written or oral quizzes at intervals.

d. Students led to discover new principles by suggestions and quizzes on the old.

e. Each student working independently, going as fast as he is able. This may apply to the entire course, to each topic or to each day's work.

f. First part of the hour devoted to the class as a whole; second part given to individual work.

g. Subject treated from the laboratory standpoint; most of the work being done in the class room.

Replies:	I.	II.	III.	IV.	V.	VI.	VII	F.
a	62	18	20	17	11	12	7	774
b	78	40	12	9	8	4	5	916
c	30	24	29	18	12	8	9	632
d	16	23	29	17	20	7	6	543
e	7	7	11	11	13	22	28	271
f	40	37	31	30	11	4	2	820
g	11	19	8	16	17	21	20	391

Several considered any definite answer to this question impossible. The objections were that (1) these modes are so intertwined with each other as to be nearly inseparable, thus making it often necessary to use several in one recitation, (2) that different treatment was necesary for different classes and (3) for different kinds of work. The result of the letters was that certain definite combinations are quite feasible, and often adhered to quite rigorously by some instructors for not only a single recitation but are used as a general plan of conducting classes. Some of those mentioned were *"a, b* and *d," "a* and *f."* In a few cases one or two double periods per week were given to work conducted along the line explained in *g*. In a few of the letters *e* and *g* were considered faddish and worthless. On the other hand quite a number of instructors who could not use these modes because of too large classes nevertheless ranked them first and considered them to be "ideal for small classes." No particular objections and no special variations in modes depending upon subject matter were expressed.

Comments:

"I depend on circumstances."

"*g* and *e* desirable but generally not practicable."

"*c* and *d* can easily be overdone, also *g*."

"I generally combine *b* and *f*."

"*a* and *f*. Engineering students seem to do best if they put problems and proofs on the board and explain daily."

"*e* is ideal if one instructor can be assigned to each three or five students."

"I would favor *a* and *f*. It is a waste of time to try to explain or lecture to the class until they have spent considerable time in the preparation."

"We are trying some experiments which might be classed under *b, g,* and *f* but have not yet gone far enough to draw any inference. To gain good results I believe it absolutely necessary that the instructor should be able

to give individual attention to the students in his charge, and this not only once a month but every hour they come to his class room or laboratory."

"The laboratory, wherever possible, is a most effective way of clinching points made in the recitation room and of clearing up points that are vague. A few hours' work in a laboratory having proper facilities will cause a student to better grasp the real meaning of 'moment of inertia' than many hours of class-room work. Laboratory and class-room work should always be worked conjointly."

"My own mode is a combination of *g* and *d*. I am one of probably very few teachers who can say, I have taught school twelve years and have yet to conduct my first recitation. I am not sure my method could be applied to the teaching of mathematics, but I surely hope such is the case. I would rate *e* first if it were practically possible. We have faithfully tried it—failure."

VII. GENERAL NEEDS AND DEFICIENCIES.

Besides the needs and difficulties pertaining to any particular topic as considered in question 2 to 10 there are those of a more general nature as the power to think mathematically, or ability to analyze problems. These are considered in the next four questions.

1) GENERAL MATHEMATICAL ABILITY—(*i*) *Need.*

19. Number 1, 2, 3, etc., in order of importance the following pertaining to the qualifications of the engineering student who has completed the course in freshman mathematics:

a. Skill and accuracy in computations.
b. Analysis of problems.
c. Interpretations of results in solution of problems.
d. Knowledge and use of equations.
e. Representation of physical laws by means of graphs.

Replies:	I.	II.	III.	IV.	V.	F
a	93	41	37	26	12	804
b	122	52	17	6	9	880
c	41	66	63	25	7	715
d	23	35	42	56	32	525
e	13	19	30	50	75	406

Of the general mathematical qualifications enumerated in the question, the tabulated results show conclusively that *a, b* and *c* are considered the chief ones, with special emphasis on *b,* the analysis of problems. It is in this latter that the most difficulties are also found, as will be seen under *20.* The replies indicate that the trouble students have with the application of mathematical theory in problems is a case of knowing "how to do" while not knowing "what to do."

Comments:

"*a* and *b* are useless without the others. They are of prime importance."

"It is my opinion that students invariably show themselves most deficient in analysis of problems, that is, in transferring the plain English statements of a problem into mathematical form, in shape to be handled by the mathematical processes with which they are often familiar."

"A practicing engineer seldom goes back to fundamentals, but works largely with standard tables and formulas, and his mathematical training is only of advantage to him to enable him to use these readily and intelligently, and to understand the history and origin of these so that he can apply them without fear of misusing or abusing them as non-technical engineers are likely to do. If an engineer has the time and opportunity to go into research work, he may then call into play his theoretical mathematics. It is well to train young engineers for that possibility, but the great majority have little opportunity for it. The mechanical branches algebra, analytic geometry, calculus and descriptive geometry and graphics, are, however, of the utmost value in the way I have indicated, and a great deal of drill work should be given in these branches and plenty of problems of as original a character as possible. It is of more importance to cultivate in the student the power and ability to attack a problem directly and intelligently and to solve it by short and sensible means than to learn many theories."

"I find difficulty in arranging the order. *c* is the whole thing from my standpoint, if one means by that the understanding of the theory in its application to examples under that theory. I mean the grasp by the student of the fundamental meaning of the subject under consideration. That is the large thing. If the student does that he can be pretty well trusted to get those minor details, the important formulas, the methods of solutions of problems and all that. *a* enters very largely in trigonometry otherwise would not be of vital importance, though accuracy should always be given a prominent place. In comparison with my remark under *c* the topics under *a, b, d* and *e* are of absolutely minor importance; still all four of these are quite important and ought to be insisted upon rather largely."

(*ii*) *Deficiencies.*

20. In which of (19) do students who have completed their freshman year show the greatest deficiency?

Replies:	*a*	*b*	*c*	*d*	*e*
	68	93	70	19	11

Comments:

"They can not apply principles to practical work."

"Usually students show a greater weakness in analysis than in many other topics and as a result, the accuracy, the interpretation, etc., are likewise deficient."

"As an example of trouble in mathematics the following may be of interest. I gave this problem to 40 senior engineers: A steam boiler when tested shows an efficiency of 65%. After providing it with an economizer it shows 85%. What is the percentage of saving of fuel? Half of the class gave the answer in this fashion $\frac{85-65}{65} = 30.75\%$ and the other half $\frac{85-65}{85} = 23.53\%$. Only one of the 40 gave a logical statement to show why the formula he used was right."

"As regards *a*, one great deficiency in all students is a lack of knowledge in regard to what degree of accuracy is called for in any calculation. They have heard a great deal too much about accuracy, and not enough advice has been given as to what is good enough for the given problem, or how to determine the required degree of accuracy."

"The weak point in the work of engineering students in the upper class courses is usually their inability to apply mathematical analysis to practical problems, and to formulate mathematical expressions for physical facts. To my mind, the remedy for this difficulty lies in emphasizing the physical rather than the philosophical aspect of mathematics, and of attaching to every mathematical expression and operation in a problem, a definite, clear-cut physical meaning. The teaching of physics and mechanics as parallel courses with the calculus is a very useful method of accomplishing this result. The constant use of easy practical problems, in which the student can thoroly grasp the physical theory of the problem, is very useful in every kind of mathematical teaching."

(*iii*) *Remedy.*

21. What is the remedy?

The remedies proposed for difficulties met under *a*, may be briefly summarized as follows: always to demand accurate results

in any computation, to regard checks upon all results as a part of the solution of the problem, and to urge upon all teachers of secondary mathematics the necessity of insisting upon checking.

The suggestions for *b* and *c* can not be presented in any such definite form as for *a*. Insistance upon analysis of problems from the grades up, the following of every theory learned with illustrative applications, and the elimination of work by formulas and rote as far as possible, are strong points. Practically no suggestion for improvement was offered regarding *d*, and nothing was said regarding *e*.

Comments:

"More concrete problems to awaken interest."

"Better and more attractive texts and more efficient instructors."

"Make students do the work rather than instructors."

"The remedy is to be found in further training not in the freshman year."

"Application and interpretation of theory; many problems."

"More drill and better training in English."

"Begin with public school work and drill, drill, drill. Pretty hard in one freshman year to undo years of slovenly habits and lack of thought."

"Before working a problem before the class give a thorough analysis of the principles involved."

"Frequent exercises in analysis by use of problems slightly different. Avoid monotony and sameness. Make the men defend each position and make good every assertion."

"Learn to read equations in analytic geometry."

"Most decidedly more training in the use of what they are getting in mathematics."

"Constant comparison and reference to actual practical work. Let student remain out of school for a year and devote himself to some real practical work under a boss."

"Require a rigid checking system."

"Special work in computation in connection with course in trigonometry."

"More drill in fundamentals and add course (*13*)."

"Require accuracy from beginning of the mathematical course."

"Insist on accuracy as of equal importance with other items."

"Have students work many problems and never use a key or answers."

"Give problems for home solution along the line needed."

"I realize that it is practically impossible in a four year course to teach everything, but for the engineering graduate who expects to make a financial success in his after life and not simply remain an entirely theoretical man and therefore always on a salary, an employee of someone

else, it has always seemed to me that if the minds of the students could be turned a little bit more toward the practical end of the work, in addition to their theoretical ideas, it would in the end be greatly to their advantage."

2) Relative Importance of Topics.

22. Designate by 1, 2, 3, etc., the order of importance for any mathematical topic of:

a. Mastery of theoretical matter involved.
b. System and neatness.
c. Accuracy in computed results.

Replies:	I.	II.	III.	F.
a	159	20	25	642
b	34	67	113	349
c	59	108	37	420

It is clear that the majority have placed the above in the order *a, c, b.* Where these were said to be of equal importance all were given the rank I.

Comments:

"Without accuracy all are worthless."
"By no means should there be any differentiation here."
"They are so interdependent it is difficult to give any answer. System and neatness are necessary to insure accuracy."

VIII. ENGINEERING vs. A GENERAL EDUCATION.

23. Is it true, as has been said, that engineering students apply themselves better than those taking a literary or scientific course?

Replies: Yes 125
No 45

Many said that as they had not had experience with both classes of students they considered themselves as incompetent of expressing any opinion on the question. This has reduced the number of replies somewhat.

Comments:

"At some institutions it is, at others not; in general it is true in the east but not in the Mississippi valley and the west."

"Yes, in mathematics; much better than the average student in literary or scientific courses, but not nearly as well as the best."

"I think it is true that the engineering students apply themselves better than those taking literary or scientific courses but am not sure that their course is the more thorough. There are scientific students who do their work much more thorough than the average engineering students, but the average engineering student has a definite idea of what he wants and he seeks those practical results."

"I believe that the training derived from an appropriately taught thorough engineering course is fully the equivalent of that obtained from a general science or literary course. In general, the grade of teaching done in engineering schools is inferior to that done in classical schools, and the student is not carried to that point in his work where the training is of its highest developmental value. For instance, emperical machine design, taught without the foundation of applied mechanics, is utterly without developmental value, rational machine design has great value along such lines. When taught for the purpose of developing the power of analysis in a certani branch of useful knowledge, engineering courses are good. When taught for the sole purpose of turning out men capable of accomplishing certain useful tasks in an orthodox and correct manner, engineering courses are of absolutely no developmental value. The first method of engineering education tends to turn out keen, clear, logical minds, capable of applying their power to any task, whether it is a problem in thermodynamics, in machine shop practice, or in public finance. The second method tends to turn out men of immediately available but limited utility, able to fill but a narrow place in the life of their community, and having an education but very little better than that acquired by the majority of skilled laborers."

IX. SUMMARY.

Replies to the questionarire letter were received from instructors of mathematics and of professional subjects, and from practicing engineers. A majority of the replies favor entrance examinations in mathematics for all students while some very grave objections are raised to such a practice, but there is a general demand for better preparation, especially in algebra. A short review, together with tests, has been found a good substitute in several institutions. Fundamental principles and the elements of the subjects taught are considered of primary importance, and in these are also found the

greatest deficiencies, as remedies for which greater intensity of work and less extension in subject matter, together with a vitalization of the work are advised. The object of analytic geometry is to furnish the student with a new mathematical language rather than information concerning any particular class of curves, and the introduction of the elements of the calculus in connection with analytic geometry is favored in a majority of the replies. Inability to analyze problems and to use the mathematical language learned are the chief general difficulties reported, and frequent application of the principles learned and teaching for future use may be helpful as remedies. It is deemed advisable to teach engineering students in separate classes, making some use of problems met in actual engineering work for the purpose of creating interest, but special texts are not strongly favored. There is a demand for work in computation, with especial attention to degrees of accuracy. Little attention is paid to questions concerned with modes of instruction, showing the need of professionally trained teachers. Engineering students are thought to be more thoro and painstaking than literary students in general.

CHAPTER V.

PRESENT NEEDS AND TENDENCIES.

I. CONTENTS OF CHAPTER.

Various questions regarding the needs of the work in mathematics for freshmen engineers and the trend of thought of the present day concerning it have arisen in different connections in the preceding chapters. The discussion of such questions has been held in abeyance until the present chapter when each can be taken up in its entirety in the light of the accumulated information. Besides the statements found in the previous chapters, use will be made of the few works published on the subject, articles in the various journals, proceedings of meetings of societies, personal interviews and personal observations. As far as possible the various suggestions for improvement have been tested by the writer's own experience.

II. PREPARATORY MATHEMATICS.

If, as has been said, a man's education should begin with his great grandparents, then surely the mathematical education of the student of engineering should begin, at the very least in the high school. The general demand for better mathematical preparation of the students entering the freshman year makes the question of preparation an essential part of our discussion.

1) Nature of Work Needed.

The preparatory mathematics of prospective engineering students should be taught with a view to its utilization. Especially is this true of algebra, where each principle should be studied for its future use, all the work being grouped around the equation as the

over

predominating feature. The work in geometry may profitably be made more observational, and a greater number of practical problems introduced, especially in solid geometry.[1] Special classes in high school mathematics for prospective students of engineering are impracticable but all the work should be given with a view to its possible application. The suggestion that the collegiate mathematics be given from the standpoint of utility applies even more forcibly to preparatory work.[2]

(*i*) *Algebra to be Treated as a Study of Equations.*

The writer had the opportunity, a few years ago, of teaching beginning high school algebra simultaneously to two sections of about equal industry and ability. With one the text was followed quite closely and the results showed the usual average of interest and achievement. The other section, as an experiment, was given an extended course in equations, all other work being taken up as necessary adjuncts. The pupils were introduced to the equation through problems of arithmetic and were shown (not told) that algebra is only an extension of arithmetic. This was done somewhat in the following manner: two or three problems were given on the relation between the sides of the right triangle which had been learned in arithmetic, for example, finding the height of a tree which had been blown down in such a maner that it is still in contact with the stump, which is six feet high, and the end reaches the ground eight feet from the foot of the stump. Then a problem was introduced regarding a tree broken off in a similar manner but where the unknown number had to be used implicitly, as where the length of the tree and the distance of its top from the base of the stump were given, the length of the part broken off being asked for. Where the results were whole numbers some solutions were always obtained, but for other cases none could be found. This brought out the necessity of more general tools for the solution of problems and the need of the "unknown quantity." At first the equation was used only to solve for the unknown quantities. These were never represented by X, Y, or Z but by L for length, W for weight, etc. After some time the second great use of the equation was taken

[1] Chap. IV, p. 81.
[2] Chap. IV, p. 81.

up; namely, to show the relation between various quantities. The graph in two variables was now introduced to give a geometric picture of this relation, and the connection between the graph and the equation established. A number of simple graphs were made either for quantities which could be read directly from the instruments presented before the class, as from levers, the apparatus exhibiting Boyl's Law, etc., or for those which could be easily computed, as for interest, change of United States to English money, etc. A little later this was extended so as to include any available data, taken mostly from physics. Two graphs drawn with the same coordinate axes furnished an introduction to simultaneous equations, where, of course, only linear equations were at first considered. Only one or two illustrations as to mode of introducing the various topics ordinarily taught in algebra can be given. Fractions were taken up when literal quantities were met with in the denominators of fractions, radicals when such equations as $X^2 = 8$ had to be solved, and later on $\sqrt{X-4} - 3\sqrt{X+2} = \sqrt{X-1}$, etc. The modes used to introduce the various topics may have been very artificial but they secured the desired results; they created a great interest in the work and produced at the end of the year a class capable of using algebraic equations and of interpreting results obtained from its solution in a very satisfactory manner.

2) Deficiencies Found.

The teaching of geometry has elicted relatively light criticism.[3] How far this is due to good teaching and how far to the fact that the future work is not largely dependant upon geometry can not readily be ascertained. But in algebra deficiencies abound; the principal ones relating to surds, exponents, quadratics, imaginaries and the manipulation of algebraic symbols. The one chief objection, however, to the high school algebra as now taught is that the students can not *apply it*.[4]

3) Cause of Deficiencies.

There are really two causes of deficiencies; those due to the curriculum and the school management, and those due to the teacher.

[3] Chap. IV, p. 71.
[4] Chap. IV, p. 91.

(*i*) *School Management.*

The school management is responsible for four great causes of poor preparation:

Fully 75% of the difficulties are due to the long interval between the work in algebra in the high school and that of the freshman year.

Students are not always required to attend classes to obtain the credits presented for entrance. This arises in two ways; first when students take all of the work in algebra at an unaccredited school, and then go to one which is accredited without having to "prove" their grades at the latter, and secondly, where a student is permitted to do part of the work outside of the regular class and "pass an examination" on the work so as "to graduate with his class," or for some similarly trivial reason.

In some cities work in mathematics, especially algebra, is alloted to various teachers, who have, as a rule, been selected for their ability in other subjects rather than in mathematics.

The last hindrance to be mentioned is that if the high school is too small to have a physical director, the mathematics teacher is often the foot-ball and base-ball coach. One never hears of a teacher of Latin selected because he can coach the athletic teams.

(*ii*) *The Teacher.*

Teachers who devote all their time to teaching mathematics may still have poor preparation or may feel little enthusiasm in their work, and will consequently produce poor results. Most of the work is taught as a set of heterogeneous topics bearing no relation to each other, and with the expectation that each will be "placed upon the shelf" as soon as "completed." In no way is the work given with a view to future use. No wonder that the students can not apply radicals or find factors when solving quadratic equations. On the other hand, an enthusiastic mathematician is liable to make the work too theoretical and to over emphasize the analytic side of the subject. In either case the pupils' interest, the one chief element of success in secondary work, is not secured.

4) Suggested Remedies.

The engineering colleges can aid in the crusade for better teaching of preparatory mathematics by demanding utilitarian mathemat-

ics from the high schools and by showing its educational value. To produce results these demands must be more than mere suggestions or requests.

(*i*) *Requirement of Entrance Examinations in Mathematics.*

One way of making this demand is to require entrance examinations in mathematics of all students.[5] Besides necessitating better work on the part of the high schools, such examinations would result in reviews of the high school subjects before taking up the collegiate work and would also acquaint the instructors with the students' weaknesses.[6] To the objection that this requirement would not be fair to the other departments, it may be said that this system is in force with the Department of English in well recognized institutions. To the added objection that a great deal of disturbance among the accredited schools would result, the reply may be that this disturbance might have a salutary effect.[7]

(*ii*) *Substitute for Entrance Examinations.*

To obtain the beneficial results of entrance examinations without their objectionable features some institutions have hit upon the plan of devoting the first two or three weeks to a review in algebra interspersed with tests.[8] For those who are seen to be notably deficient a course in the high school is recommended, and to those only slightly so, further work in review is given. This plan gives the students the necessary review and saves them from the loss of time and the discouragement of failure. All reports showed that this plan has proved a decided success wherever tried.

(*iii*) *Demands upon the High Schools.*

Engineering colleges that admit graduates of high schools without examinations should insist that the instruction in mathematics in these schools be given by special teachers of that subject, and in no case by teachers selected because of their ability as athletic coaches or on other irrelevant grounds. All work for which

[5] 64 per cent of the answers to the questionaire letter favored entrance examinations in mathematics. Chap. IV, p. 71.

[6] Chap. IV, p. 71.

[7] Chap. IV, p. 71.

[8] Chap. IV, p. 71.

credit is given should be taken in class. A review of algebra during the fourth year should also be one of the requirements for exemption from entrance examinations.

III. COLLEGIATE MATHEMATICS.

With respect to the collegiate mathematical instruction of students of engineering we consider 1) what should be taught, 2) how it should be taught.

1) What Should be Taught.

Future use should be the basis of the mathematical curriculum. Practicing engineers and instructors of engineering subjects are best qualified to speak of the future mathematical needs of the engineering student. Consequently, they may justly claim the privilege of designating the contents of the mathematical curriculum. Such suggestions from the professional men should allow for time to give all prerequisite topics, and should be really as well as nominally mathematical, calling for not the mere teaching of "rules of thumb" but for the use and application of mathematical principles.

2) How It Should be Taught.

When the matter to be taught has been determined it is the province of the mathematical department to consider the designated material and all prerequisite topics without extraneous matter, and to organize it into a teachable course. The duty of teaching this course to the very best advantage then also falls upon the mathematical instructors.

3) Specific and General Needs and Difficulties.

At the completion of the first year's work in mathematics the students are expected to show mathematical ability of two distinct kinds, specific and general; while the difficulties encountered also fall under these two heads. An example of the first would be the handling of radicals; one of the second, the ability to think mathematically. This is a helpful distinction, especially in bringing out

the opinions of educators and professional men interested in these questions, and accordingly was adhered to in making out the questionaire letter of Chapter IV.[9]

The specific requirements and difficulties are somewhat the easier to discuss and will be taken up first. It will also be found that the general ones can often be reached thru the specific and disposed of while taking care of the latter.

4) Specific Needs and Difficulties.

The specific needs and difficulties will be considered under the subjects of trigonometry, algebra, analytic geometry and the calculus both to expedite the discussion and also because most of the institutions observe these distinctions. Pedagogic questions relative to the improvement of the work will be considered in appropriate connections.

(*i*) *Trigonometry.*—a) Reason for Place in Curriculum.

Since only 18% of the institutions require trigonometry as a preparatory subject it can not be omitted in considering the work of the freshman year.[10] Further, the subject is of the utmost importance in engineering work and while comparatively easy, it presents an astonishing number of failures, and that in the elements.[11] For these reasons it requires the very best of instruction and, as a consequence, some institutions which admit without examination in other mathematical branches require one in trigonomery.[12]

b) Needs.

A thoro grasp of the meaning of the trigonometric functions and their use in the solution of the right triangle is really the one thing needed.[13] Besides this the student needs to know some few relations existing between the functions, to be able to solve a right triangle placed in any position, to pick out the triangles in a con-

[9] Chap. IV, pp. 67-70.

[10] Chap. II, p. 43.

[11] An instructor in railroad engineering said he had "some young men who have been considered as very bright in the calculus but who can make no headway because they have not the foundations of trigonometry. They have no grasp of the trigonometric functions and their relations."

[12] Chap. III, p. 54.

[13] Chap. IV, p. 74.

figuration essential to his problem, and to solve oblique triangles by use of formulas. Advanced topics as hyperbolic functions, De Moivre's Theorem, etc., are of little value to the engineering student.[14]

c) Deficiencies.

The most marked deficiencies are found in the topics just mentioned as of most importance. This is probably due to the fact that the elements are called into play the most frequently and points out the need of special care in teaching the fundamental definitions and properties of the trigonometric functions. The use of identities, especially those relating to functions of 2 X and ½ X, inverse functions, circular functions and application of the definitions of the trigonometric functions are also found to give serious difficulty while still another grave deficiency is the inability to see what triangles are to be solved and to solve them in positions other than the standard one with one side horizontal.[15]

d) Suggestions for Improvement.

In consequence of the deficiencies found, the work is to be simplified and confined mainly to the elements. The functions should be defined from the ratio standpoint and the definitions constantly reviewed. To a great number of students who have learned the functions with reference to the unit circle the sine means the vertical line opposite an acute angle in a triangle, etc. The meaning of the trigonometric functions can be made clear by the solution of numerous right triangles, which should be placed in various positions and not in any standard one. Constant application of theoretical results to a variety of real problems, of which a rich store lies at hand, is also indispensable.

e) Trigonometry to be Taught from the Utilitarian Standpoint.

The utility of no study is more obvious to the engineer than that of trigonometry. Use can be made of this fact to arouse interest and action on the part of the student. The very first lesson is the best place to bring out this utilitarian idea in order to make a lasting impression and this lesson can be made to contain a large

[14] Chap. IV, p. 73.
[15] Chap. IV, p. 74.

part of the elements of the subject. This should not be done by the formal statement that trigonometry will be a most necessary part of the future work in mathematics and physics but by well selected illustrative problems, such as finding the height of buildings, flag poles and trees by using lines of known length drawn from their bases along the ground, width of rivers using base lines along their banks, etc. If a surveyor's transit is not available for these first lessons a model should be improvised. As another means to keep up interest a formal problem where solution will be within reach of the students at the end of two or three weeks' work can be proposed on one of the first days to be kept constantly in view for solution. The finding of the resultant of several forces passing thru a point by first resolving them along two directions at right angles may be cited as an illustration. For the solution of problems on the right triangle similar to those mentioned above only the sine and tangent functions are necessary. The functions of any angle having been defined the students may be required to determine the values approximately from measurements, or they may be provided with tables of natural functions at once. The instruction should be continued in the same spirit with the solution of the oblique triangle as the ultimate goal.

f) Suggested Course.

In accordance with the above it is advisable to devote two hours per week during the first half of the year to trigonometry. The work is to be introduced by the solution of a great number of practical problems solvable by means of the trigonometric functions applied to the right triangle. This is to be followed by exercises on the relation between the functions, and also the functions of two or more angles, leading up to the solution of the oblique triangle. Logarithms are taken up just before the oblique triangle. A short discussion of circular functions should also be included.

(*ii*) *Algebra.*—a) Status in Engineering Colleges.

Of all the mathematical subjects in the engineering curriculum, algebra is in the most chaotic condition. The entrance requirements vary from nothing at all to three full years, with a corresponding variation in the collegiate work. The difference which is most to

the point is found in the topics given under the head of College Algebra. No schematic exhibition of the subject matter can be made as many of the catalogs do not state what topics are included under College Algebra and those which do, present too great a variety for comparison. The courses seem to have in common chiefly the two topics of the Binomial Theorem and the theory of equations, (the latter a very indefinite term, since its scope is rarely stated fully), and the failure to mention any review of previous work even in quadratics. As to the rest there is a wide range of variation for which no definite cause can be assigned. While some topics are more especially useful to students taking a particular course (such as imaginaries for the electrical engineer), this specialization is not recognized by institutions offering several professional courses. Neither can the reason be that algebra is given as a culture study. The many serious difficulties encountered in algebraic reductions by students in advanced mathematical or technical work make it impossible to regard algebra otherwise than as a most vital foundation for subsequent work.

b) Needs and Difficulties.

The greatest difficulty is experienced in the handling of the fundamental algebraic operations, with which, first of all, the student must be familiar. Determinants, permutations and combinations, and series are of minor importance. On the other hand, graphical solutions of equations, graphical representations of certain series, the treatment of imaginaries and the solution of equations of the third degree call for special attention.[16]

c) Suggested Improvements.—*1*) *Teach Elements.*

The first aim is to give to the student a working knowledge of fundamental principles. Every effort should be made to improve the preparation given by the high school[17] but where students are still deficient remedies must be applied at the earliest possible moment so that they may not feel a handicap later. The review of the main principles of the high school algebra as previously outlined will be found helpful.[18]

[16] Chap. IV, p. 76.
[17] Chap. V, p. 101.
[18] Chap. V, p. 102.

II) Principles of Algebra to be Taken Up When Needed.

It is a mistake to conduct the instruction on the assumption that students know and remember every principle previously studied. Perhaps the professional men ask too much of the instructors of freshmen mathematics by taking too much for granted, while the latter in turn require too much of the high school teachers. Time can often be saved by giving preliminary reviews of principles involved in new work to be taken up. The plan has been carried so far as to give no formal course in college algebra but to take up each algebraic topic as it arises in connection with other branches of mathematics. This somewhat novel idea offers some promise of help in alleviating difficulties which confront the teacher of freshman mathematics.[20] The objections to such a method are first, that it may have a tendency to foster "a rule of thumb" way of treating each mathematical process as applicable to but a single case; and second that the student does not learn the mathematical principles but only a way of connecting some mechanical manipulation with the particular process in question. To avoid so great an evil the method must not be carried to extremes, and the principles should be applied in several different phases of use. In small sections taking very specialized courses this plan commends itself most favorably. Other plans of a more radical nature are those in which college algebra is taken in connection with either trigonometry or analytic geometry in such a way as to bring out its points of contact with these subjects, and its graphic aspect.[21] As the portions of college algebra that are really advanced are little used before taking up mechanics, real college algebra can well be deferred until after analytic geometry and the calculus have been taken, the algebraic work given earlier being essentially a review and an extension of the elementary work.

(*iii*) *Analytic Geometry.*—a) As a Mathematical Language.

Since engineering students must use algebra, and as long as there exists such extreme difficulty in applying it, collegiate work in algebra should not be given them as a culture study. Let the

[20] Chap. III, p. 57.

[21] Frederick L. Emory, Advanced Algebra in Engineering and other Colleges, Proc. Soc.. for Prom. Eng. Ed., vol. III, p. 104.

instructor teach that which will be met with in the future work; for example, in radicals, let him drill on a systematic use of the three fundamental rules to which all can be reduced:

a) the removal of radical denominators,

b) simplification of expressions where the number under the radical is a perfect power indicated by a factor of the index, as $\sqrt[4]{25}$,

c) simplification of expressions containing factors of which the radical root can be extracted, as $\sqrt{12}$.

If the students afterwards offer a result such as $\sqrt{8}$ in the solution of a problem he deserves a reduction in grade.

The following few rules are quite effective in a course in college algebra:

a) give a short review of exponents, radicals, quadratics, etc.,

b) introduce new topics by oral quizzes on the earlier principles that will be involved,

c) insist that principles previously learned be employed whenever possible,

d) demand an analysis of each problem and an understanding of each formula used, without, however, requiring the development off-hand.

Chapter III contains several suggestions for making both the position and the object of college algebra definite in the curriculum.

(*iii*) *Analytic Geometry.*—a) As a Mathematical Language.

When the student takes up the study of analytic geometry, whether designated by that formal title or not, he meets with a new language just as much as when he begins the study of French or German. There must, of course, be some medium to which to apply this new language while learning it. Until very recently the conic sections have been universally selected as this medium and often the learning of the properties of the conics has been the main business of the course, thereby turning it into a mere memorization of a collection of isolated properties and formulas of the curves and losing track of the analytic mode of attack, which should be its chief aim.[22]

[22] "Too much space should not be given to the investigation of the properties of the conic sections,——etc." Mansfield Merriman, Teachers and Text-books in Mathematics in Technical Schools, Proc. Soc. for the Prom. of Eng. Ed. vol. II, p. 95.

It is wholly immaterial what medium is used so long as it supplies the requirement of teaching the language, and interests the students in its use. If in addition to this it can acquaint the student with some curves met with in his future work, that is an additional advantage. Any problems in mechanics or geometry possessing these qualifications are suitable, and conics are to be treated only as varieties of curves in general.

b) Illustrative Problems.

The following are a few illustrations of problems of the nature just mentioned which have been found useful by the writer:

1) Find the equation of a projectile given an initial horizontal motion.

2) Ditto if thrown at an angle to the horizontal.

3) Prove that parallel rays can be brought to a focus by a parabolic reflector.

4) Proof of the method of inscribing a parabola in a rectangle as used in mechanical drawing constructions.

5) Ditto for hyperbola.

The following are examples of curves for study:

$y = ax$

$y = ex$

$y = \sin(x)$

$y = (x-2)(x-1)$

$y = \dfrac{ax+b}{cx+c}$

$y = ae$

$y = \log(x)$

$y = \cos(x)$

$y = (1.5)^x$

$y = \dfrac{1}{x-3}$

$y = \sin(x+b)$

$y = \tan(x+b)$

$y = \cos(x+b)$

$y = e^x$

$y = \dfrac{10}{1-e^x}$

c) Other Devices.

It is in the application and in the appreciation of the analytic language that the most frequent difficulties arise.[23] Besides the problems mentioned above several devices have been found useful, one of which is the analytic proof of theorems found in Euclidean geometry. Such problems as the following may be assigned as a part of the following day's work:

1) "From the middle point D of the base of a right angled triangle ABC, DE is drawn perpendicular to the hypotenuse BC. Prove that $BE^2 - EC^2 = AB^2$."

2) "CO is the line drawn from the center of a circle to any point of the chord AB. Prove $OC^2 = OA^2 - AC \times CB$."[24]

For off-hand solutions in the class room the most efficient are the simpler ones involving the principles at hand, such as, to prove that the perpendiculars from the vertices of a triangle upon the opposite sides meet in a point.

For bringing out the true analytic spirit the "hammer and tongs process" may be used in which the unknown relations are given analytic expressions however long these may be, and then applied as tests to the theorem to be proved. In illustration the problems above stated may be cited. The expressions for the quantities in the formulas are worked out and with these the truth of the statement verified or disproved. Such work is often very long but is of great assistance in bringing the student to see the power of the analytic expressions of geometric facts.

A full understanding of the idea of parameter is a decided aid to a clear grasp of problems in which loci are found by eliminating a parameter from two or more equations. The parameter should be taken up early and its meaning for any particular equation discussed some time before the loci problems depending upon its elimination are attempted, as otherwise the idea of the parameter is lost in following the mechanical process of elimination. The parameter is also useful in illustrating families of curves and in showing their variations. In aiding a class to see the deductions of the equation of a locus when the parameter was a point (X', Y') the writer hit

[23] Chap. IV, p. 77.

[24] Sanders, Elements of Plane Geometry. New York, 1901. p. 216.

upon the mechanical device of using red chalk for marking the locus in the graph and also for the corresponding X′ and Y′ coordinates in the equation of the required locus. At least 90% saw at a glance the relation between the graph of the locus and its equation. In fact, before the solution was fully written out, the young man who had asked for it exclaimed, "I could have done it if I had had colored pencils." The above incident gives an excellent illustration of the value of the maxim "Teach through the eye."[25]

d) Needs and Difficulties.

The principal difficulties naturally arise in the same topics as do the greatest needs. Besides the one great need of learning the analytic language a very few special ones may be found, of which loci problems, intersection of curves, tangency and the sketching of curves from the equation ought to be mentioned.[26] Regular practice in curve sketching and the use of equations in their simplest form with slightly varied terms like the following has been found helpful to remedy defects in the latter case:

1) $y^2 = 12x$
2) $x^2 = 12y$
3) $y^2 = -12x$
4) $x^2 = -12y$
5) $y^2 = 12(x-3)$
6) $x^2 = 12(y+3)$

(*iv*) *The Calculus.*—a) Principles Taught.

Of late years the elements of the calculus have been introduced more or less into the freshman work by various institutions.[27] The general idea is to acquaint the student with the methods of the calculus rather than to enter into the theory to any extent; the formal course being delayed until the second year. Principles are developed by problems and graphs, and the methods of the calculus are used before being rigorously proved.[28] The work is, as a rule, limited

[25] J. W. A. Young, The Teaching of Mathematics in the Elementary and the Secondary Schools. New York, 1907, p. 110.
[26] Chap. IV, p. 78.
[27] Chap. III, p. 65.
[28] Chap. III, p. 53.

to problems on tangency and on maxima and minima, the former of which affords a natural means for its introduction.

b) Advantages.

This early introduction to the use and methods of the calculus and an acquaintance with its powerful machinery, is expected to meet the demand for the calculus in practical problems. It gives the most powerful and the most suitable instrument for solving a great number of problems that arise in algebra, analytic geometry and physics, in the first year or early in the second year. Further, the calculus will not only be thus applied more extensively but will also be studied during a materially longer period of time, which is a decided advantage. The interest of the student is also materially enhanced when the work is directed along the lines of what can be done with the calculus rather than with the development of formulas and their rigorous proofs. Such an introduction of the calculus should make the student feel that it is not a wholly new and distinct branch of mathematics but only a continuation and natural outgrowth of what has preceded. While the data previously recorded do not show any marked tendency actually to make this early introduction to the calculus, yet the trend of opinion is really in its favor since all who reported having tried the plan had found it a success.[29]

(*v*) *Combined Courses in Algebra, Coördinate Geometry and Elements of the Calculus.*

Coördinate geometry may be made the basis of a combined course in algebra, coördinate geometry and the elements of calculus to occupy two or three hours the first half and four or five hours the second half year. After a two or three weeks' review of the most needed preparatory algebra, the graphic representation of the algebraic function is taken up, where such problems as finding points of intersection are solved and such elementary notions as determining the distance between two points are developed. Algebraic principles are thereafter taken up as occasion demands. A full treatment of the right line and circle follows. A study of any suitable interesting curves, including conics, completes the work in plane analytic geometry, where the chief aim is to familiarize the student

[29] Chap. IV, p. 80.

with this new mathematical language and to train him in its use. Special stress is placed upon loci problems throughout. The methods of the calculus are to be used in the tangent problems, the finding of maxima and minima of functions, and all other problems which are best treated by such methods. No introduction to the calculus other than the application of its elementary principles to problems is to be attempted. Differentiation and integration are to be carried on simultaneously. If time permits geometry of three dimensions may complete the course, but this can be delayed with advantage until the study of partial derivatives.

(*vi*) *Work in Computation.*—a) Mode of Presentation.

The need of drill in computation is evident,[30] the only question being as to the mode of presentation. Two modes suggest themselves: one, the formulating of such work into a special course as has been done by some institutions,[31] and the other its distribution throughout the mathematical course. While the former plan would emphasize the necessity of such drill the latter is rather to be preferred as it will show the necessity of accuracy at all times and enhance the value of the drill by extending it over a much longer period. Besides the assignment of special exercises in computation as occasion arises it has been found to be quite helpful to require all final expressions to be reduced to their simplest form for computation; that is so as to involve the minimum amount of arithmetical operations. For example, let radicals always be reduced to the form $\frac{a}{b}\sqrt[n]{c}$ which is the simplest for computation by the use of tables. Let the student employ tables, even though unacquainted with the theory involved in their construction.

By far the most potent aid in achieving accuracy is the application of checks upon the work, which is always possible except in rare instances. To impress the need of such checks make them a part of the problems assigned and to emphasize it still further change the data of the problems in the text or assign outside ones in which accuracy is the main requirement. The importance of this phase of the problem work for students of engineering can not be overesti-

[30] Chap. IV, p. 85.
[31] Chap. III, pp. 62-63.

mated. An added advantage of no mean importance gained from such a system of checks is the drill in insight and ingenuity which it gives.

b) Degrees of Accuracy.

Of nearly equal importance with accuracy itself is the ability to understand the degree of accuracy of which a problem is capable. It is worse than useless to permit a student to carry out computations to decimal places beyond those for which the data are known to be correct. The three following problems, selected from different books on trigonometry, may serve as illustrations showing how a much greater degree of precision is demanded in the result than is found in the data if the latter are to be considered as having the degree of precision ordinarily met with:

1) The height of a house subtends a right angle at a window on the other side of the street; and the elevation of the top of the house from the same place is 60°. The street is 30 ft. wide. How high is the house?

The result is given as 69.282 ft. This result, professing to express the height of the house within a thousandth of a foot, implies that the width of the street has also been measured with this degree of accuracy. This is not customary and not even readily possible. To do so it would be necessary that the sides of the street themselves be fixed within a possible error decidedly under a thousandth of a foot, and that measuring instruments of corresponding delicacy be employed in measuring the width of the street. In practice these conditions are never fulfilled. A good question would have been to ask for the height in feet to within an error of 1 ft. or of 1 in.

2) From the top of a hill I observe two mile-stones on the level ground in a straight line before me and I find their angles of depression to be respectively 5° and 15°; find the height of the hill.

The result, given as 228.6307 yds., professes to give the height of the hill within one ten-thousandth of a yard, or within about 1/250 in., which is much less than the presumable error in the placing of the mile-stones, and which also implies that the height of the

hill is a quantity that can be determined to within the diameter of a grain of sand.

3) A ladder 40 ft. long reaches a window 33 ft. high on one side of the street. Its foot being at the same point, it will reach a window 21 ft. high on the opposite side of the street. Find the width of the street.

The result is given as 56.649 ft., to which the same criticism is applicable as to problem 1).

When accuracy to two decimal places is meant such expressions as 46 in. ought never to be allowed but 46.00 in. insisted upon. Both from the engineering standpoint and from that of general mathematical accuracy these conditions should be considered in all computations.

5) General Needs and Difficulties.

Several needs and difficulties of a general nature have already been considered in connection with the various branches of study. The following are further general phases which have not been touched upon:

(i) Coöperation with the Professional Departments.

The success of the instructor of mathematics can be greatly enhanced if the instructors of the professional branches will, from time to time, supply him with lists of mathematical difficulties encountered by their students and with the suggestions as to the preparation needed for their respective courses. It is not meant that the department of mathematics should duplicate the work of the professional departments in any sense, but only supplement it so as to increase the efficiency of the work of both departments. It is not meant to teach a special "rule of thumb" for each particular thing to be taken up in the future but that the principles and results that will be necessary later are to be emphasized, in precisely the same way as the instructor of trigonometry emphasizes certain principles and results to be used later in the calculus, mechanics, etc. In this day of correlation it seems surprising that definite steps have not been taken to any great extent in the direction just mentioned. The special courses in mathematics outlined in Chapter III have

been arranged more or less according to this plan, it is true, and one is based altogether upon the work to be taken in the professional departments during the last two years but the plan has received no wide recognition. Surely such coöperation of the departments would prove greatly beneficial by increasing the interest of the students in their mathematical work thus having its service and utility pointed out, by remedying much of the weakness of the students in applying their mathematics to the work of the professional courses, and finally by fostering more congenial relations between the mathematical and professional departments.

(*ii*) *Problems.*—a) Their Use.

Every principle studied must be illustrated by a sufficient number of problems, for the purpose of fixing it in the minds of the students and of clearing up any difficulties which may have arisen as to its application. To emphasize and fix the mathematical language many short problems should be given for immediate solution during the class period: the shorter the problems and the greater the number, the better.

b) Kinds of Problems.

Problems are like puzzles: if too simple they do not hold the attention and if too difficult they extinguish all ambition to solve them. Long and complicated problems, such as those of the old "fox and hound" found in elementary algebras only a few years ago, ought to be discouraged. Simplified explanations with a large number of simple problems will greatly aid in preventing students from doing things by rote and from imitating imperfectly some sample problem which has been worked out in the text-book. In order to interest the students the problems should not be fantastic but related to their every day life as far as possible. For the purpose of creating further interest the use of problems connected with engineering work have been suggested.[82] There is, of course, danger of overdoing and great care must be exercised in the selection and in the preparation of such problems. They must not be "fake" engineering problems, nor too technical nor contain too many elements. To make such a problem usable the useful parts may have to be selected

[82] Chap. III, pp. 59-61. Chap. IV, p. 83.

and put into such form that the student can comprehend it. On the other hand the data should not be given in such a manner as to make the solution a mere matter of substitution in a formula.

c) Problems to Develop Mathematical Thought.

The student who can not think mathematically has at best a superficial knowledge of the mathematical language and consequently is unable to apply it. "Eternal vigilance" on the part of the instructor is necessary to see that his students are getting a grasp of this one essential and are really thinking the problems out and not using a "sleight-of-hand" trick for doing each set of problems. A good illustration along this line is furnished in the study of asymptotes. The student should not be permitted to write down the equation of the asymptotes of $4y^2 - 9x^2 = 36$ as $2y \pm 3x = 0$, off-hand unless he can find the values of M and K which will make $y = Mx - K$ an asymptote. To insure that he really does this, he can be required to find the asymptotes of $4x^2 - 6xy + 4x + 6y - y^2 = 10$, of an ellipse or of some curve as $y(2a - x) = 4ax$. Avoid such text-books as differentiate the classes of problems too highly, and such as introduce each set by some one worked out in detail, leaving nothing for the student but the substitution of numerical values. When a problem has been solved vary the conditions just a trifle and test the class on the new aspect.

6) Pedagogic Considerations.

The following questions on the pedagogy of the subject are of a general nature, not connected with any particular topics or principles and hence will here be discussed separately.

(*i*) *Modes of Instruction.*

It is not supposed that any instructor will hold to any one mode to the exclusion of all others, nor to any fixed combination at all times. Some combination of the previously mentioned modes is, however, generally followed by each instructor.[33] But even though an instructor may have been very successful with the use of one mode still that might not be the one with which he could be the

[33] Chap. IV, pp. 88-90.

most successful. Nor does it seem likely that the instructor will intuitively select the mode best suited to each case. The selection consequently deserves careful study.

(*ii*) *Suggested Combinations on Modes.*

The following combinations of modes are recommended as the most useful:[34]

A) Assignment of work with or without explanations and the laboratory mode.

B) Heuristic and laboratory modes.

C) A modification of the heuristic mode applied to the class as a whole during the first half of the period, the second half being devoted to the individual students while working problems. This is in the main as follows: the work for the following day is assigned first, avoiding hurry at the end of the period and incidentally giving an incentive for prompt attendance. The theoretical matter for the day is then taken up in the form of an oral quiz, in connection with which short illustrative problems are solved. This is intended to clear up points of difficulty, to fix important matters and to insure preparation from day to day. Whenever possible the fundamental principles of the next assignment are taken up in connection with the quiz on the lesson of the day and are worked out by the class with slight suggestions from the instructor. Should it be impossible to connect the future work with that of the day, the class is reviewed on past work with which it can be connected. About half of the hour is consumed in this manner. The remainder of the time is given up to individual work by the students on problems either at the boards or at their seats, preceded when necessary by a short general discussion on particular difficulties in the day's problems. During this work on problems the instructor "visits" each student, quizzing him and giving him suggestions. Individual teaching is approached as much as the size of our present day classes will permit. As the instructor is dealing with the individual he has even more freedom to select the mode "best suited to the case" than if he had

[34] Chap. IV, p. 88.

to consider the class as a whole, but is, of course, more limited as to time. One of the real dangers is the temptation to slight the good students in favor of the mediocre, or vice versa. A college class of from twenty to twenty-five and a high school class of from twenty-five to thirty can be handled successfully in this way.

(*iii*) *Special Texts in Mathematics.*

The texts used in mathematics are of various forms and degrees of specialization. A first class consists of those which treat each topic separately and with little connection with the succeeding material or future application, but lay special stress upon completeness of the proofs and logical sequence. As a rule the problems are abstract and play a minor part. A second class is not nearly so profuse in proofs but still preserves the sharply separated topics. The problems are not so abstract but often impossible or too hypothetical. This is what may be termed the usual old standard form of texts. The third class confines itself to definite topics, but not so fully as the two previous classes, and these topics are selected with a view to their future use as the primary object. Points of special interest to engineers are brought out, as use of the slide rule, degree of accuracy in computations, etc. For the most part the problems are simple, real and sensibly shorn of the fantastic. Those of the fourth and last class are constructed along quite radical lines. They disregard the divisions of algebra, analytic geometry and the calculus and form a homogeneous whole of the three for use over a period of from one to two years. Often these are written for the needs of the students at some particular institution or of a particular man, and as such may not be generally useful.

There is quite a division of opinion as to the advisability of using texts of the fourth class. In several instances where highly modified courses are given, texts of the second or third class, supplemented by problems and additional material of a more decided engineering nature, are used.

The early introduction of the ideas of the calculus already discussed (p. 111) is quite feasible under such a plan. The methods of the calculus that are used would be developed in the class by the

teacher as needed, and it is quite possible that the student would thus appreciate these new appliances better than if he were introduced to them in a more formal way.

(*iv*) *The Profession of Teaching.*

The seemingly prevalent idea that no attention need be given the question of instruction must be abandoned before results of the desired excellence are obtained.[35] Surely the history of mathematical teaching in this country, and especially in technical work, does not show it to be so simple that the unprepared beginner can do the work properly. It is gratifying to note that attention is being directed to this most vital question and it is hoped it will soon receive attention commensurate with its importance.[36] The reason for the present excellent instruction in Latin is the fact that the teachers of Latin are trained to teach; that the pedagogy of the subject is one of the main things dwelt upon; that the Latin departments of our advanced institutions give "teachers' courses" not only for prospective instructors of secondary schools but also for those of collegiate grade.[37] One of the absolute requisites for good teaching of mathematics at engineering colleges is the proper preparation of instructors as teachers and the placing of mathematical instruction on the plane of a profession. This applies especially to teachers of freshmen who are often without wide experience, while the mathematics given in the freshman year is largely the foundation of the whole professional course.

[35] Chap. IV, p. 89.

[36] In a recent address Prof. Alexander Smith, of Columbia University, said that life is a series of lessons in "problem solving" and that no subject was fit for the college curriculum which could not be reduced to that basis. Continuing he said: "If the American College is to be rehabilitated along these lines, namely, those of teaching problem solving and giving the work of professional standards, or along similar lines, two important improvements in the present situation are required. To give all subjects the kind of instruction indicated will requre skillful teachers, and it is generally admitted that the teaching of undergraduates is at present less satisfactory than is that of the pupils in the grades and high school on the one hand, or of the student of the graduate school on the other. The second need is that of more efficient methods of teaching, particularly in non-linguistic subjects." Alexander Smith, Rehabilitation of the American College, Address before the Educational Sec. Am. Chem. Soc., July 1909. Science vol. 30, p. 457. See also E. H. Moore, Presidential Address on "The Foundation of Mathematics." Bull. Am. Math. Soc. 1903, p. 402.

[37] E. H. Moore, Presidential Address, l. c.

John Perry, Preliminary Education of Engineers, Address before Engineering Section Brit. Soc. for Prom. of Sci., School Science and Mathematics, vol. II, p. 264. 1902.

V. SUMMARY.

The present chapter has been reserved for the discussion of the various questions which have arisen in connection with the data previously recorded. For good work during the freshman year it is necessary that the students come well prepared in the secondary work; especially in algebra. Two plans have been proposed to insure such a preparation. The first is to require entrance examinations in mathematics of all students. The second is to admit without examination in mathematics only students coming from high schools which have special teachers of mathematics and which give a half year's work in algebra the fourth year. The following is a suggested curriculum: trigonometry for two or three hours per week during the first half of the year, and a course in algebra and analytic geometry which begins with a review of preparatory algebra for the first two or three weeks, and is taken two or three hours per week the first half year and for five hours per week the second half year. Analytic geometry is to be the basis of this course; topics of algebra being taken up as needed or as they arise in connection with the other work of the course. The object of analytic geometry is the acquisition of a new mathematical language and not information regarding any particular set of curves. The elements of the calculus are brought into use in the solution of the tangent problems and in the finding of maxima and minima of functions. As much mathematics is to be taught the students in the first year as can be included in the curriculum without overcrowding, and they are thus to be given, at the earliest opportunity, the most powerful weapons for attacking the problems they may encounter later. The mathematical curriculum is to embody such principles as the instructors of the technical subjects consider necessary for the future work, and is to be taught by men qualified as instructors of mathematics.

CHAPTER VI.

CONCLUSION.

I. CONTENTS OF CHAPTER.

The present chapter restates briefly the most salient results of the investigation recorded in what precedes, and presents the conclusions drawn therefrom.

II. ORIGIN AND GROWTH OF THE ENGINEERING COLLEGE.

All engineering education in the United States has been the result of a demand created by economic conditions. When the apprenticeship system proved inadequate to produce sufficiently trained engineers, graduates from the Military Academy were drawn into professional work along civil lines. Stephen Van Rensselaer, one of the early promoters of American industrial enterprise, gave substantial and timely aid to the cause of industrial education by the founding, in 1824, of the Rensselaer School. This school was the pioneer institution of its kind and in 1835 offered the first course in civil engineering in the United States. Soon afterwards several literary institutions organized courses in civil engineering and industrial chemistry, which at first were mostly modifications of the last year or two of the regular literary courses. By the middle of the ninetneenth century industries had arisen that required men with technical education in mechanical as well as in civil engineering. This condition prompted Congress to pass the Land Grant Act of 1862 as an aid to mechanical colleges. Since that time various courses have been added as occasion demanded, such as Electrical Engineering, Sanitary Engineering, etc. As a member of our educational system, the engineering college is only fifty years old.

III. PROGRESS OF WORK IN MATHEMATICS AND PRESENT NEEDS.

In our survey of the progress made in the mathematical work of the engineering colleges up to the present time and in a discussion of future improvements we have considered three phases: The preparation, the collegiate curriculum, and the problem of teaching the courses demanded.

1) ENTRANCE REQUIREMENTS.—(*i*) *Their Scope.*

During the formative period, or until the coming of the Land Grant Colleges in the early sixties, all the engineering courses offered, with the exception of those at the Rensselaer School, were modifications of the last year or two of existing literary courses. Accordingly two or three years of literary collegiate work were required for admission to these technical courses. As more and more work was added to the technical courses the amount of the literary collegiate work required for admission to the technical work had to be reduced until when the complete four years' technical course was offered the only work common to the two courses was that for entrance. The door was then opened for differentiating the entrance requirements, and this has also been done to some extent. In mathematics they have been raised for the technical courses from arithmetic (prior to 1850) to the present average of one and one-half years of algebra, one year of plane geometry and one-half year of solid geometry. Trigonometry is required by 18% of the institutions.[1]

(*ii*) *Present Needs.*

The above entrance requirements are at the present adequate as to extent. The need is rather for more concrete presentations of the subjects and with a view to their applications, and for more emphasis upon the principles that will be met in the collegiate work, especially the principles concerning the handling of algebraic expressions and their application to problems, in both of which many deficiencies are found. This demand that preparatory algebra be

[1] Chap. II, p. 43.

taught concretely and for its applied value, that the various topics be treated as parts of a homogeneous whole centered about the equation, that fundamental principles be made to stand out boldly and clear from the working machinery involved, and that the work throughout be emphasized by numerous fairly simple problems. Such teaching of mathematics for future utility is conceded also to be the most generally valuable from the educational point of view.

(iii) Remedy.

How to secure the good preparation in entrance mathematics that is so vital to successful work in mathematics during the freshman year, is one of the most serious questions confronting the engineering colleges of today. Unless the high school teachers have been especially trained for the teaching of mathematics, satisfactory preparation of students is impossible. It is the duty of the engineering colleges vigorously to advocate such training and to require entrance examinations in mathematics of the graduates of all high schools not having a special teacher of mathematics. In addition to this, only such high schools should be accredited as give a half year's review of algebra in the fourth year. As a final means of securing good preparation for the work of the freshman year two or three weeks' review at the beginning of the year is recommended, and students then found especially deficient should be required to take additional preparatory work.

(iv) Usefulness of Variation in Entrance Requirements.

The great difference in entrance requirements[2] found among the various engineering colleges seems, at first glance, a condition to be deplored. Yet if the high schools take advantage of this divergency they will find it of inestimable help in aiding those of their students who intend taking an engineering course, to select an institution commensurate with their capacity. Strangely enough, the great number of failures in mathematics on the part of freshmen students makes it appear as though little cognizance were taken of this divergency.[2] The lack of uniformity in entrance requirements may also serve as the entering wedge for the organization of some system of grading our engineering colleges. When

[2] Chap. II, p 43.

this is done technical education will have been placed upon a firmer foundation and much of the present waste of time and energy will be saved, especially in the work in mathematics.

2) Collegiate Curriculum.—(*i*) *Progress.*

Until 1850 the collegiate courses included all of algebra, together with plane and solid geometry, trigonometry and a little analytic geometry given in the latter part of the second year. After that time some algebra was required for entrance and the collegiate work was increased correspondingly; calculus was first added in 1855, but came late in the course and was little used in the technical work. Since then the time devoted to analytic geometry and the calculus has been materially increased, and these subjects have been placed early in the curriculum so as to make them available for the technical work. At present from seventy to one hundred and twenty class hours are devoted to analytic geometry, which is completed during the first year or in the early part of the second year, and from one hundred to two hundred class hours are given to the calculus in the second year. It is only within the last fifteen years, however, that attention has been given to the introduction of work especially planned for engineering students, and these modified curricula are as yet only in the formative state.[3] Some of the more noteworthy features are: 1) the treatment of the subjects from the standpoint of application rather than that of philosophy, that is, less attention is paid to the rigorous proofs of theorems than to their application; 2) less insistance upon sharp lines of demarkation between the subjects; 3) the introduction of laboratory courses; 4) the correlation of the work in algebra and analytic geometry, without special attention to the conics; 5) the introduction of the calculus into the work of the freshman year, especially in connection with the solution of the tangent problem.[4] The causes for the adoption of this modified work may be summarized as a desire to lessen "the gap between theory and practice."[5]

[3] Chap. III.
[4] Chap. III, pp. 52-62.
[5] Chap. III, p. 66.

(*ii*) *Suggested Improvements.*

Ever since the founding of the Rensselaer School the object of the engineering colleges has been the production of technically trained men. The principles necessary to such a training must, therefore, control the formation of the curriculum. To no subject is this more applicable than to mathematics, so fundamental for the more technical work. Future use, not culture value, must be the criterion according to which mathematical topics are admitted or rejected. It hence becomes necessary for the instructors of the professional studies to cooperate with those of the mathematical department by designating what mathematical principles are needed in the professional courses. All mathematical topics should be treated in as close proximity as possible to their application. When the usual division into subjects is not adhered to principles which are allied to each other will naturally be connected, and thus the value of each will be enhanced. It is a decided advantage to equip the student with the most powerful instrument applicable to a problem or set of problems the *first* time such problems arise, for example, the use of derivatives in the solution of tangent problems. The most efficient means will thus become the natural one for the student to use and he will become more proficient in handling them thru prolonged application. The mathematical curriculum for the freshman year should contain as much usable mathematics as possible without overcrowding.

3) INSTRUCTION—(*i*) *Progress and Present Needs.*

The men who furnished the information under the apprenticeship system could in no wise be called mathematicians or teachers. From the coming of the engineering colleges there has been continual advancement in mathematical knowledge among the instructors until few deficiencies are at present found on that side. Some attention is also being given to the question of preparing men as teachers,[6] but on the whole little has been done in this direction as is shown by the lack of interest in the technical questions of instruc-

[6] Chap. V, p. 120.

tion.[7] Until due attention is given to this most vital phase of the problem and the work of mathematical instruction is raised to the rank of a profession, the desired results can not be hoped for.

(*ii*) *Special Phases to be Noted.*

The following are some of the salient points to be considered relative to the teaching of mathematics to freshmen students of engineering.

a) General Aims.

The general aim of the instruction in mathematics given to freshman students should be to bring out its utilitarian value. To accomplish this end it is necessary to make the relations between the principles and their application the primary object, relegating exhaustive proofs to the background. In thus learning to apply the principles the student will also secure a clearer understanding of the principles themselves than he could possibly attain thru any other means. Such a utilitarian mathematics fully understood by the student and applied by him in a logical and intelligent manner is also of the highest educational value.[8] No study should be introduced for its value as a mental gymnastic.

b) Fundamental Principles.

For the engineering student even more than for the literary student attention to fundamental principles is of the utmost importance. These principles should therefore first be studied thoroly and illustrated by a great number of slightly varied problems; and secondly, the reduction of each problem to these fundamentals should thereafter be insisted upon at all times. Such a thoro grounding in fundamentals may necessitate the elimination of some of the "advanced work," but without the ground work this would be largely superficial, while the student thoroly equipped with the basic principles will be able to do the more involved work as it arises.

c) Problems.

The value of any educational work is directly proportional to the interest taken by the students. In mathematics this interest is

[7] Chap. IV, p. 89.

[8] Such a rational study has been ably described on page 95.

largely dependant upon the problems proposed for solution. As the work of the freshmen engineers is to be given mainly from the standpoint of application it is readily seen what an important part problems play and what great care is needed in their selection. The data selected for the problems should be as real as possible and entirely shorn of the fantastic. The use of problems from engineering data is of value not only in giving the students some little insight into the nature of their future work but especially in holding their interest. Still, before being used in the mathematical work such data must be carefully edited so as to exemplify the principles which it is desired to illustrate, without requiring the student to become acquainted with the technical part of the engineering questions involved. Too long and too complicated problems, on the one hand, and those requiring mere substitution in a formula on the other, must be guarded against, and "fake" engineering problems must never be used. The instructor must also be careful not to overdo matters and must take especial pains to avoid the duplication of the work of the professional departments. Problems should never be selected for their symetry of form; such problems are rarely met with afterwards and should rather be avoided, perfectly general but simple data being used instead.

d) Computation.

Accuracy in computation is a vital question with the professional engineer,[9] and the instructors of technical subjects can rightly require some help from the instructors of mathematics along this line. This can be given either by special courses in computation, such as have been inaugurated by some institutions,[10] or what appears on the whole more satisfactory, by carrying along some work in computation thruout the whole mathematical curriculum. To carry out this plan the following few points should be observed by the instructor:

1) Never let the value of numerical accuracy be slighted.
2) Drill the students to present final results in the form best suited for computation; as $\frac{1}{3}\sqrt{6}$ in the place of $\sqrt{\frac{2}{3}}$.

[9] Chap. IV, p. 85.
[10] Chap. III, pp. 62-63.

3) Introduce the use of tables whenever they materially shorten the work and their use can be thoroly understood by the students.

4) Permit the use of formulas only after the student is thoroly acquainted with the quantities involved and the relations expressed between them. No recognition is to be given solutions based upon formulas except where perfect accuracy is obtained. Whenever a common sense application of arithmetic or the elements of algebra and geometry will suffice for the solution of a problem, the use of a formula should be discouraged.[11]

5) Teach what is meant by the degree of accuracy needed and what degree is possible in any given problem.[12]

4) Too Much Required of the Mathematical Department.

According to the statistics previously recorded[13] the language departments recieve only slightly less time than the mathematical departments while the semi-professional departments receive somewhat more than it does. As mathematics is considered to be the foundation of engineering science, is it not fair to inquire in the light of these figures if too much is not asked of the instructors of mathematics? As none of the other departments seem able to relinquish any of their portion of time, then from all experiences noted, two things are necessary: 1) the mathematical courses must be made "more intensive and less extensive" and 2) the mathematical departments must have the cooperation of the professional departments in order to make the courses in mathematics fit the demands made upon them.

III. MATHEMATICS FOR FRESHMEN STUDENTS OF ENGINEERING.

The mathematical courses for freshmen students of engineering should be composed of the topics and principles which are considered by the instructors of technical subjects to be prerequisite

[11] For confirmation and illustrations see p. 92.
[12] Chap. V, p. 114.
[13] Chap. II, p. 49.

for such subjects and this material together with what it presupposes should be formulated into a compact teachable whole by the teachers of mathematics. The curriculum is to contain as much usable mathematics as possible; only the most powerful methods of attack, those which will be required for use in the future, being taken up. Basic principles involving the fundamental conception of the subjects are to receive chief attention, but little time being given to so called "advanced work." Each principle studied should be amply illustrated by simple problems based, as far as possible, upon real data. Intelligent and accurate application of the principles rather than their rigorous proofs should characterize the work thruout. The need of accuracy in computed results and the degree of accuracy attainable should be emphasized, and the placing of results in the best form for computation should always be insisted upon. Instructors trained as teachers as well as thoroly versed in the subject matter to be taught are requisite for successful work. The mathematics for the freshmen students of engineering should be a "vitalized mathematics" dealing with as real and interesting data as possible; it should be a utilitarian mathematics taught rationally and intelligently so as to give the students power in using the mathematical type of thot as well as training in its technical application.

BIBLIOGRAPHY.

Alderson, V. C. German Technical Schools. Chicago, 1901.

Alderson, V. C. The Progress and Influence of Technical Education. Proceedings Society for the Promotion of Engineering Education, XIII: 126-45. 1905.

Armstrong, H. E. The Teaching of Scientific Methods. London and New York, 1903.

Baker, I. O. Engineering Education in the United States at the End of the Century. Proceedings Society for the Promotion of Engineering Education, VIII: 11-27. 1900.

Benjamin, Park. United States Naval Academy. New York, 1900.

Bohannan, R. D. A Calculation Blunder Common to Many Texts on Trigonometry used in Engineering Courses. Proceedings Society for the Promotion of Engineering Education, XV: 655-61. 1907.

Bohannan, R. D. A Neglected Opportunity to Teach Consistant Measurement in Teaching Trigonometry. Proceedings Society for the Promotion of Engineering Education, XV; 662-67. 1907.

Burr, W. H. The Ideal Engineering Education. Proceedings Society for the Promotion of Engineering Education, I: 1-49. 1893.

Cajori, F. Some Hints on Teaching Mathematics to Engineering Students. Proceedings Society for the Promotion of Engineering Education, XIII: 26-33. 1905.

Cajori, F. The Teaching and History of Mathematics in the United States. Government Printing Office, 1890.

Campbell, D. F. Calculus for Engineers. New York and London, 1905.

Clark, I. E. Education in the Industrial and Fine Arts in the United States, IV: 838-48. United States Bureau of Education, 1898.

Comstock, C. E. A Few Difficulties in the Teaching of Mathematics. School Science and Mathematics, 7: 12-18. 1907.

Comstock, C. E. Unification of Secondary Mathematics. Proceedings National Educational Association, pp. 519-21, 1904.

Creighton, W. H. P. Methods of Teaching. Proceedings Society for the Promotion of Engineering Education, V: 101-16. 1897.

Cullum, Bvt. Maj.-Gen. G. W. Early History of the United States Military Academy. Boston and New York, 1891.

Drown, T. F. Technical Training. Journal of Franklin Institute, 116: 329-54. 1883.

Duncan, L. Technical Education—What It Should be. Engineering Magazine, 26: 161-68. 1903.

Eddy, H. T. Engineering Education. Proceedings Society for the Promotion of Engineering Education, V: 11-25. 1897.

Emory, F. L. Advanced Algebra in Engineering and Other Colleges. Proceedings Society for the Promotion of Engineering Education, VII: 104-09. 1899.

Fletcher, R. The Present Status and Tendencies of Engineering Education. Proceedings Society for the Promotion of Engineering Education, VIII: 181-90. 1900.

Franklin, W. S. and Macnutt, B. Teaching of Elementary Mechanics. Proceedings Society for the Promotion of Engineering Education, XV: 316-34. 1907.

Goldziher, K. Mathematical Laboratories. School Science and Mathematics, VIII: 753-57. 1908.

Gould, E. S. Engineering Mathematics. Cassiers Magazine, 27: 227-32. 1904.

Groat, B. F. Teaching Calculus to Engineering Students. Proceedings Society for the Promotion of Engineering Education, XII: 225-30. 1904.

Hall and Frink. Trigonometry. New York, 1909.

Hathway, A. S. Mathematics for Engineering Students. Bulletin American Mathematical Society, VII: 266-71. 1901.

Haynes, A. E. What Should be the Characteristic Features of the Teaching of a Course in Mathematics for Engineering Students? Proceedings Society for the Promotion of Engineering Education, VIII: 308-14. 1900.

Haynes, A. E. The Justification of the Use of the Expression "Engineering Mathematics." Proceedings Society for the Promotion of Engineering Education, XIV: 127-38. 1906.

Heaviside, O. Electromagnetic Theory. Introduction in vol. I and Mathematics in vol. II. London 1893-99.

Hedrick, E. R. Approximations and Approximation Processes. School Science and Mathematics, VIII: 617-25 and 745-52. 1908.

Hollis, I. N. The Naval Academy as a Technical School. Proceedings Society for the Promotion of Engineering Education, XII: 159-92. 1904.

Jackman, W. S. Correlation of Mathematics. Educational Review, 25: 249-64. 1903.

Jackson, D. C. Desirable Products from the Teacher of Mathematics—Point of View of an Engineering Teacher. School Science and Mathematics, V: 67-74. 1905.

Jackson, J. P. Methods of Study for Technical Students. Proceedings Society for the Promotion of Engineering Education, XI: 101-16. 1903.

Kent, W. Pedagogical Methods in Engineering Colleges. Proceedings Society for the Promotion of Engineering Education, XV: 90-98. 1907.

Kenyon, A. M. Teaching Calculus to Engineering Students. Proceedings Society for the Promotion of Engineering Education, XII: 220-25. 1904.

Low, D. A. The Teaching of Mathematics in the United Kingdom; (No. 8) Geometry for Engineers. London, 1911. (Too late for use in above.)

Lunn, A. C. Outline of a Coherent Course in College Algebra. American Mathematical Monthly, XII: 123-29. 1905.

McNair, F. W. The Calculus for Engineering Students. Proceedings Society for the Promotion of Engineering Education, V: 139-49. 1897.

Marx, C. D. General Education for Engineers. Popular Science Monthly, 67: 442-46. 1905.

Merriman, Mansfield. Teachers and Text-books in Mathematics in Technical Schools. Proceedings Society for the Promotion of Engineering Education, II: 95-103. 1894.

Miller, G. A. Reform in Mathematical Instruction. Science, 24:493-96. 1906.

Moore, E. H. Cross-section Paper as a Mathematical Instrument. School Science and Mathematics, VI: 429-50. 1906.

Moore, E. H. The Foundations of Mathematics. Presidential Address at Annual Meeting American Mathematical Society, 1902. Bulletin American Mathematical Society, IX: 402-24. 1903.

Myers, G. W. A Class of Content Problems for High School Algebra. School Science and Mathematics, VII: 19-33. 1907.

Myers, G. W. Laboratory Method in Secondary Schools. School Review, XI: 727-41. 1903.

Newcomb, S. Teaching of Arithmetic. Proceedings National Educational Association, pp. 86-102, 1906.

Newcomb, S. Teaching of Mathematics. Educational Review, IV: 277-86 and VI: 332-41. 1892 and 1893.

Newson, H. B. Graphic Algebra. Boston, 1905.

Perry, J. Calculus for Engineers. London, 1897.

Perry, J. Electrical Engineering as a Trade and as a Science. Nature, 63: 41-47. 1900.

Perry, J. Preliminary Education of Engineers. School Science, II: 264-72. 1902.

Perry, J. Reform in Mathematics. London, 1902.

Perry, J. Practical Mathematics. London, 1907.

Preece, Sir W. H. Functions of the Engineer. Nature, 61:374-77. 1900.

Prout, H. G. The Future of Engineering. Scientific American Supplement, 52:21451-52. 1901.

Randall, O. E. Descriptive Geometry. Proceedings Society for Promotion of Engineering Education, XV: 617-28. 1907.

Rickets, P. C. History of Rensselaer Polytechnic Institute 1824-94. New York, 1895.

Rietz, H. L. Present Situation in Regard to Teaching Algebra in our High Schools. School Science and Mathematics, VIII: 496-502. 1908.

Ripper, W. Technical Education in Relation to Industrial Progress. Nature, 61:356-57. 1900.

Russell, J. H. Systematic Technical Education. London, 1865.

Sanders, A. Elements of Plane Geometry. New York, 1901.

Saxelby, F. A Course in Practical Mathematics. London and New York, 1905.

Schneider, H. Cooperative Course in Engineering at the University of Cincinnati. Proceedings Society for the Promotion of Engineering Education, XV: 291-406. 1907.

Slichter, C. S. Improvement of the Freshman Year in Mathematical Instruction in Technical Schools. Proceedings Society for the Improvement of Engineering Education, XIV: 146-62. 1906.

Smith, D. E. The Teaching of Elementary Mathematics. New York, 1900.

Smith, R. H. Graphics. London and New York, 1889.

Soley, Rear-Admiral J. R. Historical Sketch of the United States Naval Academy. Government Printing Office, 1876.

Spencer, H. What Knowledge is Most Worth. Collected Essays on Education, p. 84. New York, 1883.

Stanwood, J. B. Mathematics in Engineering Colleges. Cassiers Magazine, 28: 148-50. 1904.

Steinmitz, C. P. Theoretical Elements of Electrical Engineering. New York, 1909.

Sutherland, J. C. The Engineering Mind. Popular Science Monthly, 62: 254-56. 1903.

Swain, G. F. Some Observations Regarding the Value of Mathematics to the Civil Engineer and on the Teaching of the Subject to Civil Engineers. Proceedings International Mathematical Congress, p. 361. 1909.

Talbot, A. N. Requirements in Mathematics for an Engineering Education. Proceedings Society for the Promotion of Engineering Education, I: 50-62. 1893.

Thurston, R. H. On the Organization of Engineering Courses, and On Entrance Requirements for Professional Schools. Ithaca.

Thurston, R. H. Professional and Academic Schools. Educational Review, 17: 16-36. 1899.

Townsend, E. J. Analysis of the Failures in Freshman Mathematics. School Review, X: 675-86. 1902.

Trotter, A. P. Mathematics and Engineering. Engineering Magazine, 26: 266-68. 1902.

Trotter, A. P. Usefulness of Mathematics from the Engineers' Standpoint. Engineering, 76: 358-60. 1903.

Walker, F A. Place of Technical Schools in American Education. Educational Review, II: 209-19. 1891.

Warren, S. E. Coordination of Polytechnic Schools. Education, 24: 232-44. 1903.

Wertheimer, J. Higher Technical Education in Germany and Great Britain. Nature, 68: 274-76. 1903.

Wilson, E. B. Some Recent Books on Mechanics. Bulletin American Mathematical Society, VIII: 403-09 and IX: 25-39. 1902.

Woods and Bailey. A Course in Mathematics. Boston and New York. 1907.

Young, J. W. A. The Teaching of Mathematics in the Elementary and Secondary Schools. New York. 1907.

Young, J. W. A. What is the Laboratory Method? Mathematical Supplement of School Science, III: 50-56. 1903.

N-E Coast Institute of Engineers and Ship-builders (England). Technical Education in Germany. Engineering Magazine, 26: 999-1000. 1902.

Notable Addresses in England and Germany. Problems in Technical Education. Engineering Magazine, 24:267-68. 1902.

Report of Standing Committee on Entrance Requirements. Proceedings Society for the Promotion of Engineering Education, IX:263-91 and X: 195-206. 1901 and 1902.

Report of Committee on Requirements for Graduation. Proceedings Society for the Promotion of Engineering Education, XII: 99-130. 1904

Committee No IX, International Commission on the Teaching of Mathematics in the Technological Schools of Collegiate Grade in the United States. (To late for use in above.)

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